

1 **Supplement to: The generic nature of the tropospheric response to sudden**
2 **stratospheric warmings**

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ABSTRACT

21

22 1. Model

23 a. Model Setup

24 As mentioned in the main manuscript, this study uses the Model of an Idealised Moist Atmo-
25 sphere (MiMA Jucker and Gerber 2017). Our model setup for the 50-year free-running control
26 integration (CTRL) is essentially the same as that recently developed by Garfinkel et al. (2019,
27 hereafter G19), but with some slight differences in the albedo profile and prescribed q-fluxes (see
28 sections 3a and 3b, respectively, in G19 for details and notation). In terms of the q-flux profile, we
29 prescribe q-fluxes in the vicinity of the Gulf Stream and Kuroshio Current which are taken here
30 to have maximal values of $Q_{\text{Gulf}} = 100\text{W m}^{-2}$ and $Q_{\text{Kuroshio}} = 30\text{W m}^{-2}$, respectively. These
31 are larger than the $Q_{\text{Gulf}} = 60\text{W m}^{-2}$ and $Q_{\text{Kuroshio}} = 25\text{W m}^{-2}$ prescribed in G19. G19 further
32 incorporated heat transport over the North Sea to approximate observed q-fluxes, but these are
33 omitted here.

34
35 In terms of albedo, we use a latitudinal step profile as compared to a smoother *tanh* profile used
36 in G19. In particular, a baseline albedo of 0.26 (to approximate the shortwave effect of clouds)
37 in our study is increased south of 70°S to a value of 0.65 to approximate the higher albedo over
38 Antarctica. In G19 these values are 0.27 and 0.75 with a smooth transition between the two. G19
39 further include a term to smoothly increase the albedo over the Arctic, but this is omitted here.
40 Note that these differences in q-fluxes and albedo do not affect our results quantitatively.

41

42 *b. Control Run Climatology*

43 In figure 1, we present the December-February (DJF) climatologies of the zonal-mean zonal
44 wind \bar{u} , zonal-mean temperature \bar{T} and the quasi-geostrophic refractive index n^2 for the 50-year
45 control run (CTRL). Note that n^2 is calculated similarly to in the main article, except that \bar{u}
46 and \bar{T} are first averaged over DJF before calculating \bar{q}_ϕ . It is clear that the polar vortex in
47 CTRL is too strong and cold compared to that observed (e.g., Andrews et al. 1987). Such model
48 biases in the background state can be problematic in identifying SSWs using absolute thresh-
49 olds such as the reversal of \bar{u} at 60°N , 10 hPa as it can underestimate the number of SSWs (e.g., ?).

50
51 The refractive index n^2 (b; calculated for a stationary planetary wave with $k = 1$ and $c = 0$) is
52 very similar to that found in observations (e.g., Andrews et al. 1987) with a minimum near 40°N in
53 the lower stratosphere and very large values close to the zero-wind line in the subtropics. Hence,
54 Rossby waves (as identified using the Eliassen-Palm flux; climatology not shown) propagate
55 upward and equatorward, avoiding the n^2 minimum in the midlatitude lower stratosphere.

57 **2. Refractive Index Evolution**

58 This figure should be compared with the n^2 and \bar{q}_ϕ anomalies shown in figure 9 in the main
59 manuscript.

61 **3. Wave-flux Diagnostics**

62 We here provide further diagnostics of the EP fluxes in order to emphasise the linearity of the
63 tropospheric and stratospheric response to the imposed forcing, as well as an analysis of the

64 Eulerian mean momentum budget.

65

66 To understand how the wave anomalies change for different thermal forcing, figure 3a shows
67 timeseries of synoptic-wave $F^{(\varphi)}$ area-averaged over 45-55N and vertically integrated over
68 500-200 hPa, for each PTRB heating run and for CTRL. These latitude and pressure-level ranges
69 were chosen so as to approximately capture the region of anomalous tropospheric synoptic-wave
70 anomalies shown in figures 6-7 in the main manuscript. In CTRL, $F^{(\varphi)} > 0$ is found at most lags,
71 in agreement with figure 6 in the main text, although it is not always significantly different from
72 the climatology. After lag 21 (marked by a vertical dashed line), CTRL and PTRB are generally
73 similar with $F^{(\varphi)} > 0$. In general, PTRB runs with stronger thermal forcing yield larger-magnitude
74 $F^{(\varphi)}$ anomalies. Nevertheless, there is some overlap between different experiments due to internal
75 tropospheric variability. Note that the $F^{(\varphi)} < 0$ at lags 10-20 in PTRB, is related to the EP-flux
76 fountain present in figure 7b in the main manuscript which forms as a result of the induced
77 tropospheric westerlies shown in figures 2b-d.

78

79 Figure 3b shows the planetary-wave $F^{(z)}$ area-averaged over 50-80N and vertically integrated
80 over 150-5 hPa to highlight the planetary-wave suppression following the SSWs. In CTRL,
81 the preceding anomalous upward wave activity is clearly evident. After the onset, there is a
82 suppression of planetary waves for all PTRB experiments, although it is insignificant for the
83 5-K PTRB. The suppression is of similar magnitude to the CTRL $F^{(z)}$ at lags >10 , after which
84 the PTRB and CTRL anomalies evolve very similarly. The PTRB ensemble means are clearly
85 separated, with stronger thermal forcing yielding stronger planetary-wave suppression after the
86 onset date. The suppression lasts for $\sim 80 - 90$ days in all experiments.

87

88 To examine the momentum balance in the tropospheric jet shift region, figure 3c shows the
 89 Eulerian mean momentum budget:

$$\frac{\partial \bar{u}}{\partial t} = f\bar{v} - \frac{1}{a \cos^2 \varphi} (\overline{u'v'} \cos^2 \varphi)_\varphi - \overbrace{\left[\frac{\bar{v}}{a \cos \varphi} (\bar{u} \cos \varphi)_\varphi + \frac{1}{\rho_0} (\rho_0 \overline{w'u'})_z + \bar{w} \frac{\partial \bar{u}}{\partial z} \right]}^{\text{ageostrophic terms}} + \bar{X} \quad (1)$$

90 (e.g., Andrews et al. 1987) averaged over 50-65N and vertically-integrated over 700-200 hPa
 91 (chosen to coincide with the polar $\bar{u} < 0$ anomalies). It is clear that the dominant balance is
 92 between the eddy momentum flux convergence $-(a \cos^2 \varphi)^{-1} (\overline{u'v'} \cos^2 \varphi)_\varphi$ and the coriolis
 93 torque acting on the mean meridional circulation ($f\bar{v}$) with the ageostrophic terms being small.
 94 $\partial \bar{u} / \partial t$ is well approximated by the sum of the other terms. In particular, $\partial \bar{u} / \partial t$ is negative for the
 95 first ~ 35 days after which it oscillates around zero (but insignificant). Overall, this indicates that
 96 there is a divergence of momentum from high latitudes (i.e., a convergence of the EP flux) which
 97 is balanced by a poleward meridional circulation (see figure 10 in main text) in agreement with
 98 Simpson et al. (2009).

100 4. The Residual Circulation

101 a. Residual Circulation Decomposition

102 Figures 4- 5 show the same as in figure 10 in the main manuscript, except for the contribution
 103 of the $\Psi_{\bar{v}}$ and $\Psi_{\bar{v}'\theta'}$ to the full residual mean meridional circulation Ψ^* .

104 b. Stratospheric NAM Variability

105 In figure 10 in the main article, the tropospheric response of the residual mean meridional
 106 circulation Ψ^* to a SSW was a tripole with $\Psi^* > 0$ at midlatitudes, flanked at low and high
 107 latitudes by $\Psi^* < 0$. We here show that this feature is actually the tropospheric response to

108 general stratospheric NAM variability. In figure 6, Ψ^* (shading) and \bar{u} (contours) are regressed
109 (as a function of latitude and pressure) onto the NAM index in both the troposphere (a; 500 hPa)
110 and stratosphere (b; 10 hPa). Both are then scaled by the magnitude of the November-April NAM
111 index at that level, to ensure the same units as in figures 2 and 10 in the main article.

112

113 Regressed onto the tropospheric NAM (a), \bar{u} shows a dipole which approximately straddles the
114 climatological jet maximum, with $\bar{u} < 0$ ($\bar{u} > 0$) on the poleward (equatorward) flank of the jet.
115 This dipole extends into the stratosphere, peaking near 70N, 10hPa. The Ψ^* response is a dipole
116 of opposite-sign to the \bar{u} dipole in the troposphere and likely represents a change in the width of
117 the Ferrel cell.

118

119 Regressed instead onto the 10-hPa NAM (b), the tropospheric response to a negative strato-
120 spheric NAM is a \bar{u} dipole which weakly penetrates down to 700 hPa. In Ψ^* , the response is
121 $\Psi^* > 0$ in the stratosphere (indicating a strengthened Brewer-Dobson circulation as observed in
122 the onset of a SSW), and a tropospheric tripole with $\Psi^* > 0$ at midlatitudes, flanked at low and
123 high latitudes by $\Psi^* < 0$. The tropospheric response is very similar to that found in figure 10
124 in the main article following a SSW. It represents changes in the width of the Polar, Ferrel and
125 Hadley cells (e.g., Martineau et al. 2018). The relatively weak penetration of \bar{u} to the near-surface,
126 compared to that found after an SSW onset (see figure 2 in the main article), is likely due to
127 the inclusion of all negative NAM events in the stratosphere (no matter their magnitude) in the
128 analysis here.

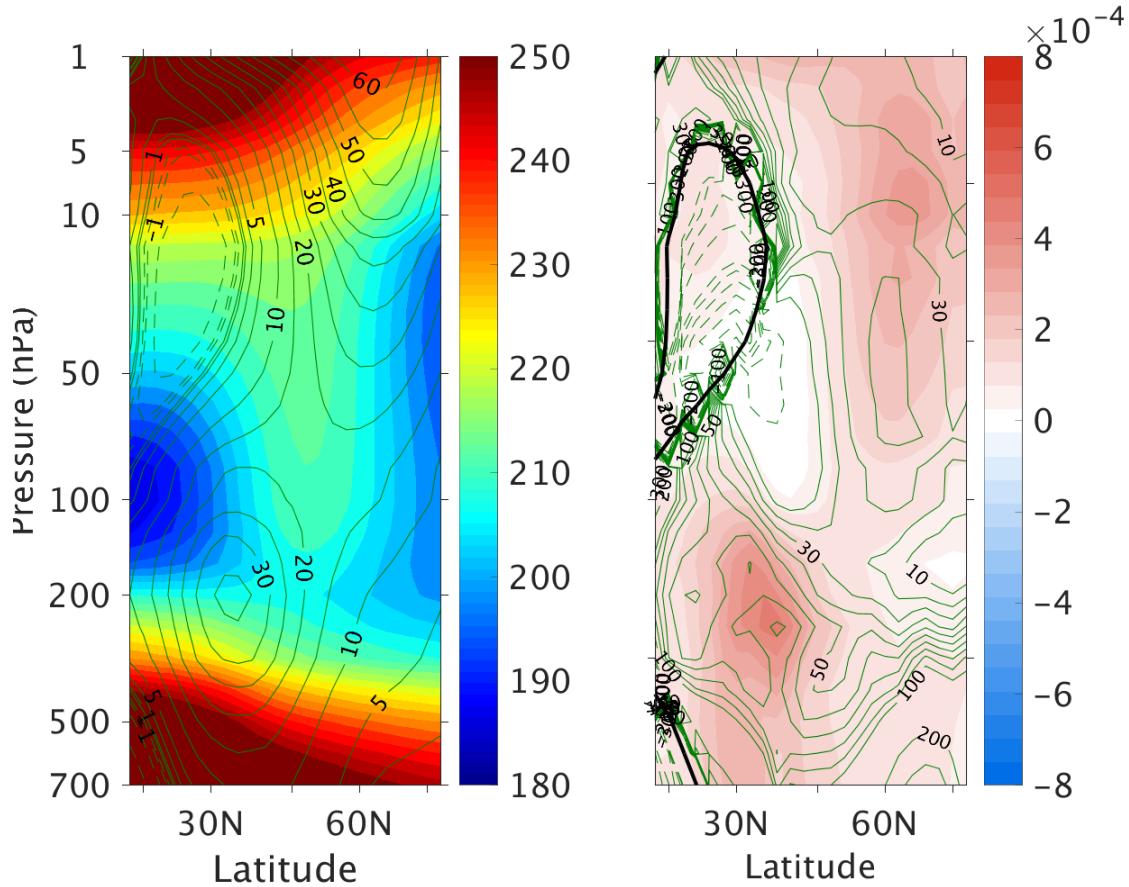
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130 **References**

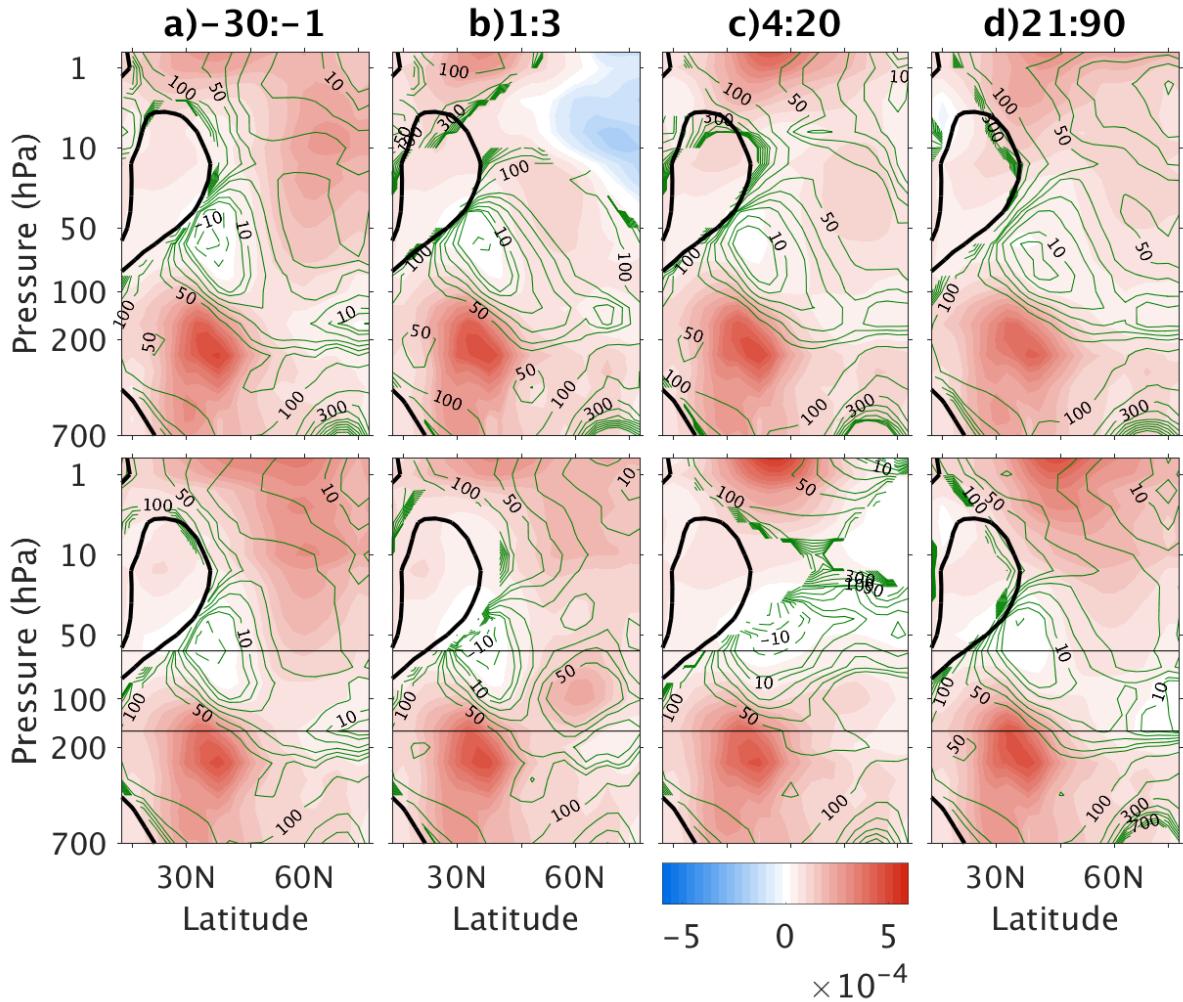
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142 response to stratospheric heating perturbations. *J. Atmos. Sci.*, **66**, 1347–1365.

LIST OF FIGURES

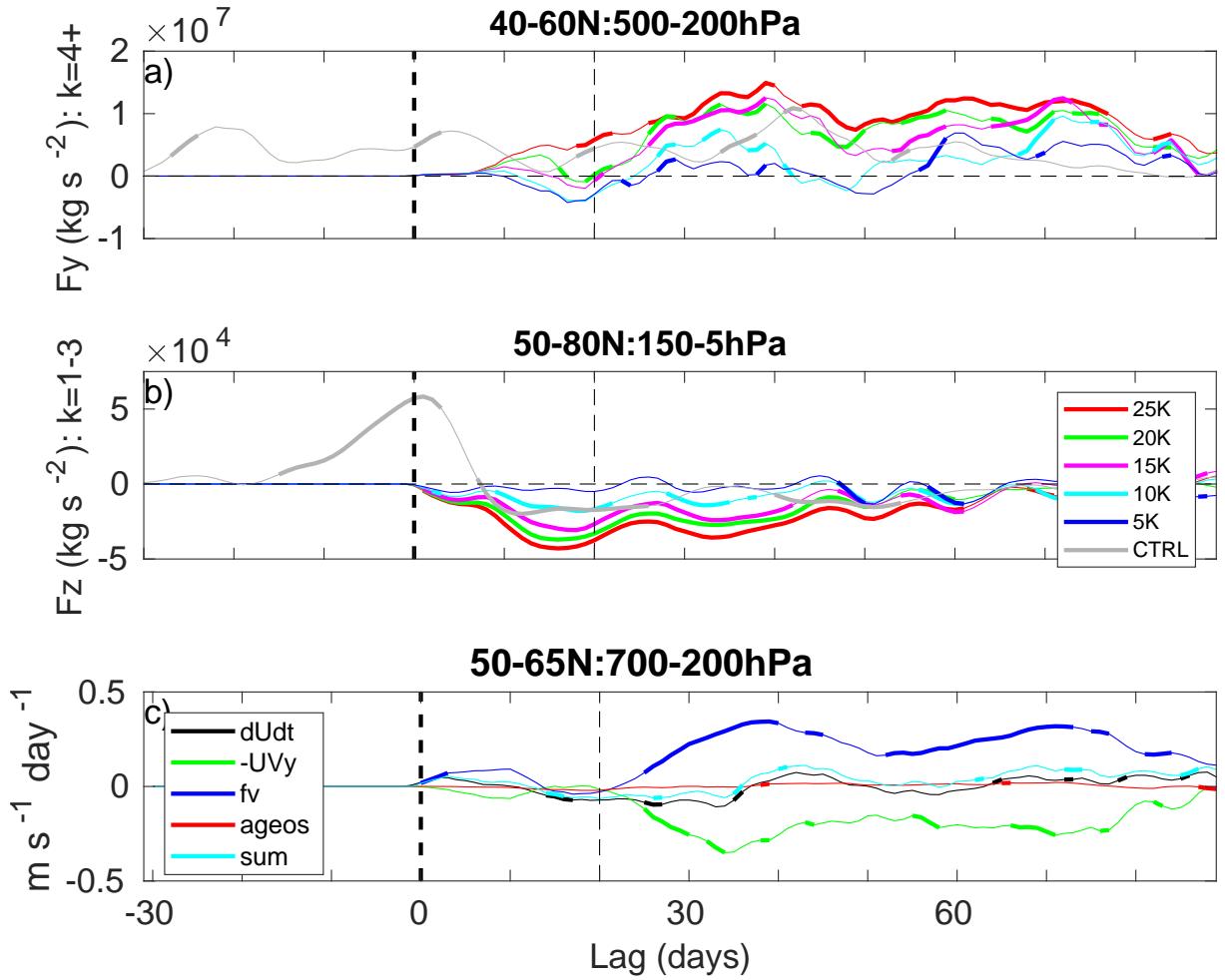
- 143
- 144 **Fig. 1.** (a) December-February climatologies of the zonal-mean zonal wind \bar{u} (contours; units of
 145 m s^{-1}) and temperature \bar{T} (shading; units of K) for CTRL. The contour spacing for \bar{u} is
 146 $\pm 1, 2.5, 5, 10, 15, \dots \text{ m s}^{-1}$. (b) Same as (a) except for the quasi-geostrophic potential
 147 vorticity gradient \bar{q}_ϕ (shading; units of s^{-1}) and refractive index n^2 (contours; dimensionless
 148 as scaled by a^2). n^2 has contours at values of $\pm 10, 20, \dots, 100, 200, 300, \dots$. Thick black line is
 149 the December to February climatological zero-wind line. 11
- 150 **Fig. 2.** Quasi-geostrophic refractive index (n^2 ; contours) and potential vorticity gradient (\bar{q}_ϕ ; shad-
 151 ing) averaged over various lag stages for CTRL (top) and the 15-K PTRB experiment (bot-
 152 tom). Solid (dashed) green contours indicate positive (negative) n^2 . Note that the full field
 153 is presented here, in contrast the the anomalies in the main text. Further, n^2 has been scaled
 154 by a^2 and is hence dimensionless, whereas \bar{q}_ϕ has units of s^{-1} . Contours of n^2 have been
 155 omitted where $\bar{u} < 0$ (N.B. that \bar{u} is the full field and not the anomaly). See main text for
 156 details regarding the calculations for both CTRL and PTRB. Thick black line as in figure 1.
 157 Horizontal lines in the bottom row are as in figure 1b in the main text. 12
- 158 **Fig. 3.** Timeseries of (a) synoptic-wave $F^{(\phi)}$ area-averaged over 40-60N and vertically integrated
 159 over 500 to 200 hPa, and (b) planetary-wave $F^{(z)}$ averaged over 50-80N and 150-5 hPa,
 160 for the ensemble means of each PTRB run and CTRL. Double-thickness lines indicate
 161 statistically-significant differences from the CTRL climatology at the 95% level. Note that
 162 (a) has been smoothed by a 9-day running mean to emphasise the salient features. (c) Time-
 163 series of terms in the Eulerian mean momentum budget (eq. 1) for the 15-K PTRB as con-
 164 tributions to the acceleration of the zonal-mean zonal wind ($\partial\bar{u}/\partial t$). In the legend, $dUdt$
 165 represents $\partial\bar{u}/\partial t$, fv is the Coriolis torque acting on the mean meridional wind ($f\bar{v}$), $-UVy$
 166 is the eddy momentum flux convergence $-(a \cos^2 \phi)^{-1}(\overline{u'v'} \cos^2 \phi)_\phi$ and $ageos$ represents
 167 the ageostrophic terms defined as $-\bar{v}(a \cos \phi)^{-1}(\bar{u} \cos \phi)_\phi - \overline{w'u'_z} - \rho_0^{-1}(\rho_0 \overline{w'u'_z})_z$. The sum
 168 is calculated as the sum of all of the terms excluding $\partial\bar{u}/\partial t$. Note that the sum is approxi-
 169 mately equal to $\partial\bar{u}/\partial t$ indicating that the budget nearly closes. Thick lines are the same as
 170 in a-b. 13
- 171 **Fig. 4.** As in figure 10 in the main manuscript except for latitude-height cross-sections of the con-
 172 tribution of $\Psi_{\bar{v}}$ (units of kg m s^{-2}) to Ψ^* , averaged over lags (a) -30-1, (b) 1-3, (c) 4-20, and
 173 (d) 21-90 for the CTRL (top row) and 15-K PTRB experiment (bottom row). Note that the
 174 two lag stages 4-10 and 11-20 in figure ?? have been averaged into a single panel here, for
 175 brevity. Note that only $\Psi_{\bar{v}}$ anomalies which are statistically significantly different from the
 176 climatology in CTRL are shaded. Black contours represent the corresponding \bar{u} anomalies
 177 at these lags with contours at $\pm 0.5, 1, 2.5, 5, 10, \dots \text{ m s}^{-1}$ 14
- 178 **Fig. 5.** Same as supplementary figure 4 except for $\Psi_{\bar{v}T}$ 15
- 179 **Fig. 6.** The residual mean meridional circulation Ψ^* (shading; units of kg m s^{-2}) and \bar{u} (green
 180 contours; units of m s^{-1}) regressed as a function of latitude and height onto the NAM index
 181 at (a) 500 hPa, and (b) 10 hPa. Note that the NAM index is calculated by projecting the daily
 182 area-weighted geopotential height anomalies (i.e., deviations away from CTRL climatology)
 183 onto the first empirical orthogonal function and then normalising by the standard deviation.
 184 The NAM index used here is highly correlated with that presented in the main text (figure
 185 3). Horizontal black lines show the level which Ψ^* and \bar{u} are regressed onto in each panel.
 186 Note the different colorbars for each panel. 16



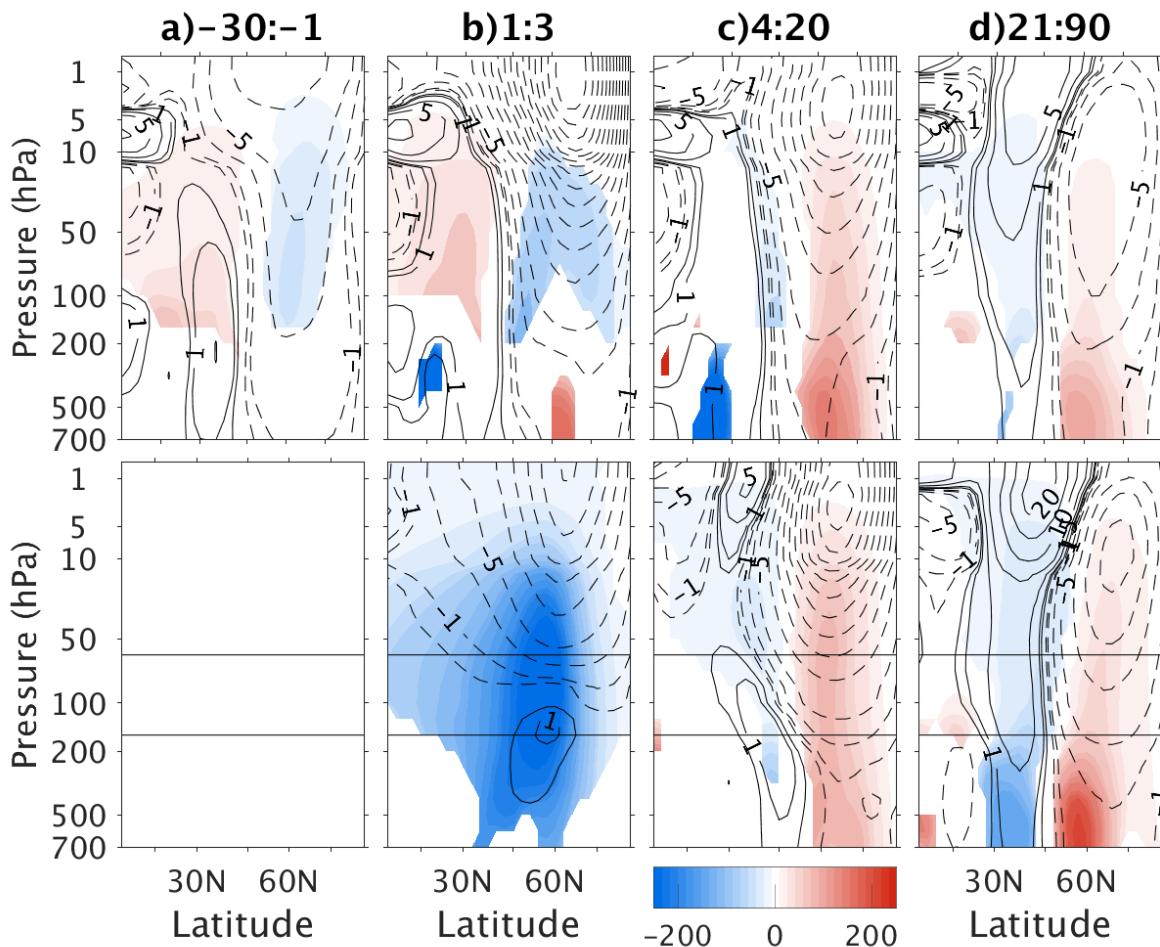
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 189 Same as (a) except for the quasi-geostrophic potential vorticity gradient \bar{q}_ϕ (shading; units of s^{-1}) and refractive
 190 index n^2 (contours; dimensionless as scaled by a^2). n^2 has contours at values of $\pm 10, 20, \dots, 100, 200, 300, \dots$.
 191 Thick black line is the December to February climatological zero-wind line.



192 FIG. 2. Quasi-geostrophic refractive index (n^2 ; contours) and potential vorticity gradient (\bar{q}_ϕ ; shading) av-
 193 eraged over various lag stages for CTRL (top) and the 15-K PTRB experiment (bottom). Solid (dashed) green
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 197 for details regarding the calculations for both CTRL and PTRB. Thick black line as in figure 1. Horizontal lines
 198 in the bottom row are as in figure 1b in the main text.



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 205 is the Coriolis torque acting on the mean meridional wind ($f\bar{v}$), $-UVy$ is the eddy momentum flux convergence
 206 $-(a \cos^2 \varphi)^{-1}(\overline{u'v'} \cos^2 \varphi)_\varphi$ and ageos represents the ageostrophic terms defined as $-\bar{v}(a \cos \varphi)^{-1}(\bar{u} \cos \varphi)_\varphi -$
 207 $\overline{w\bar{u}}_z - \rho_0^{-1}(\rho_0 \overline{w'u'})_z$. The sum is calculated as the sum of all of the terms excluding $\partial\bar{u}/\partial t$. Note that the sum is
 208 approximately equal to $\partial\bar{u}/\partial t$ indicating that the budget nearly closes. Thick lines are the same as in a-b.



209 FIG. 4. As in figure 10 in the main manuscript except for latitude-height cross-sections of the contribution
 210 of $\Psi_{\bar{v}}$ (units of kg m s^{-2}) to Ψ^* , averaged over lags (a) -30-1, (b) 1-3, (c) 4-20, and (d) 21-90 for the CTRL
 211 (top row) and 15-K PTRB experiment (bottom row). Note that the two lag stages 4-10 and 11-20 in figure ??
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 213 significant different from the climatology in CTRL are shaded. Black contours represent the corresponding \bar{u}
 214 anomalies at these lags with contours at $\pm 0.5, 1, 2.5, 5, 10, \dots$ m s⁻¹.

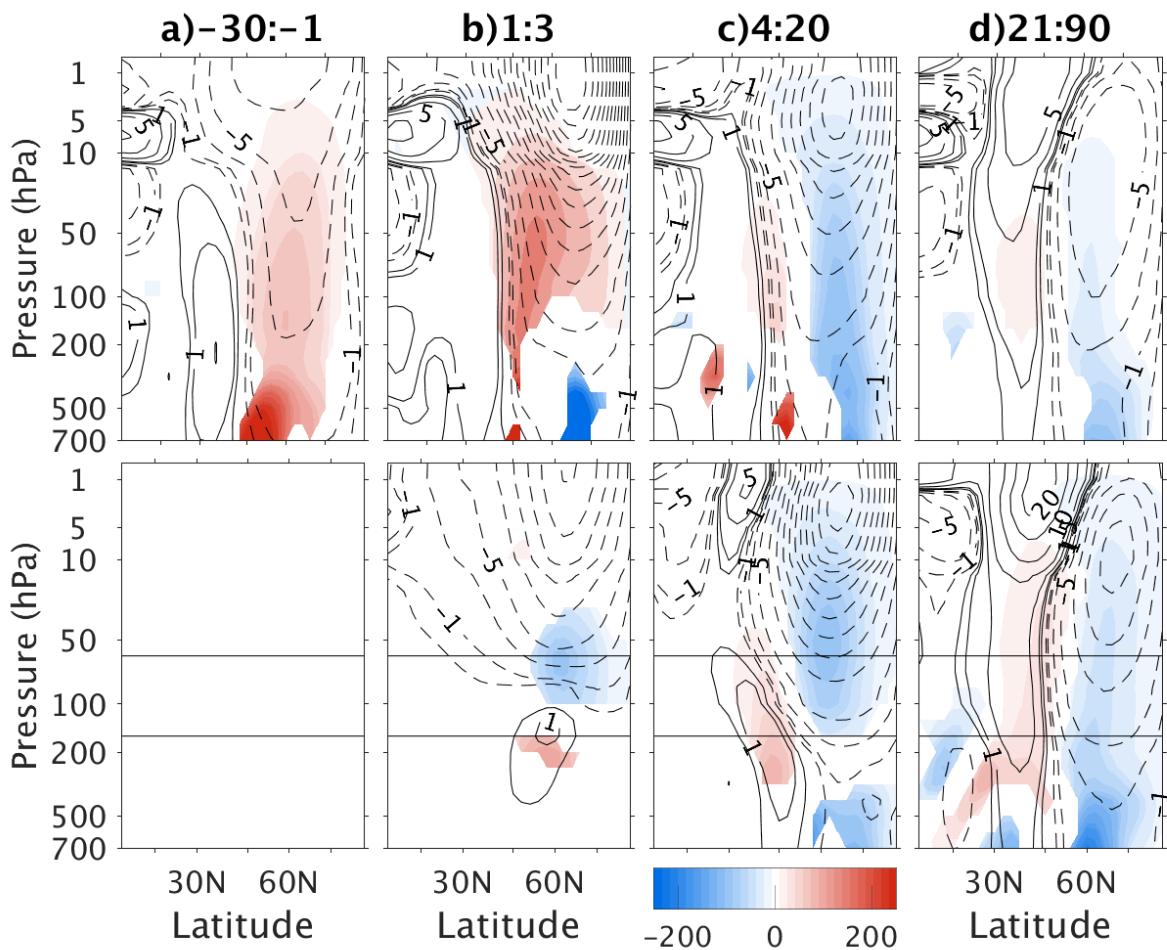
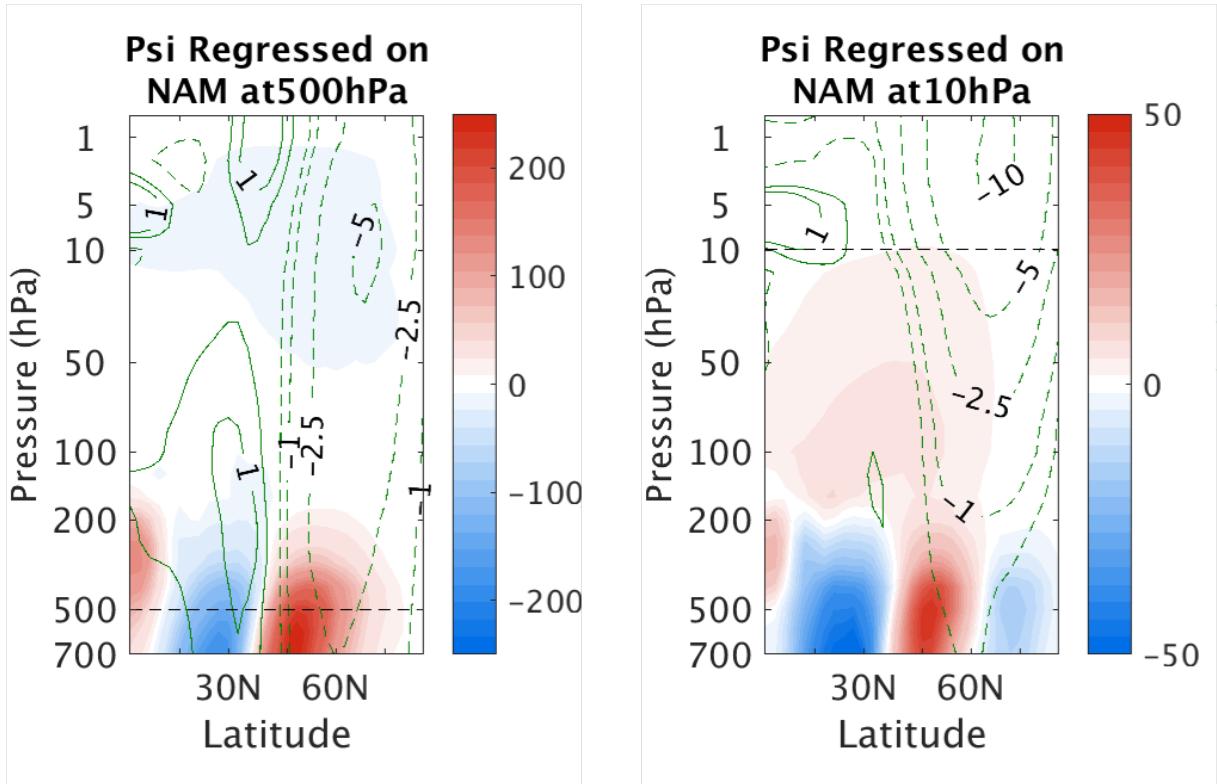


FIG. 5. Same as supplementary figure 4 except for Ψ_{vTr} .



215 FIG. 6. The residual mean meridional circulation Ψ^* (shading; units of kg m s^{-2}) and \bar{u} (green contours; units
 216 of m s^{-1}) regressed as a function of latitude and height onto the NAM index at (a) 500 hPa, and (b) 10 hPa.
 217 Note that the NAM index is calculated by projecting the daily area-weighted geopotential height anomalies (i.e.,
 218 deviations away from CTRL climatology) onto the first empirical orthogonal function and then normalising by
 219 the standard deviation. The NAM index used here is highly correlated with that presented in the main text (figure
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 221 colorbars for each panel.