A model hierarchy for data-driven gravity wave parameterization

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(1) Motivation

- A key metric of gravity wave (GW) parameterization tuning is the fidelity of the simulated Quasi-Biennial Oscillation (QBO).
- Simulated QBOs in an intermediate complexity atmospheric model (MiMA), forced with emulators of physics-based GW parameterization (AD99¹), are highly variable.



- Sensitivity analysis of the QBO response to external forces $(e.g., CO_2)$ and GW parameters is computationally taxing.
- We explore the generalization and calibration of data-driven GW parameterization in a 1D QBO model testbed.

(2) Model and stochastic wave forcing

• A hybrid of the 1D QBO models studied in HL72² and P77³, forced by a collection of monochromatic waves packets:

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - \kappa \frac{\partial^2 u}{\partial z^2} = -\frac{1}{\rho} \frac{\partial}{\partial z} \sum_i A_i \exp\left\{-\int_{z=z_1}^z \frac{\alpha(z')}{k_i(u-c_i)^2} dz'\right\}$$

• The wave spectrum follows AD99¹:

$$A(c) \propto \operatorname{sgn}(c) \exp\left[-\ln 2\left(\frac{c}{c_w}\right)^2\right]$$

• We add stochasticity to the wave forcing: at each time step the total source flux $F_{S0} = \sum_i |A_i|$ and spectral width c_w are drawn from a bi-variate log-normal distribution.



• Physically, F_{S0} (related to precipitation²) and c_w (related to convection depth) are positively correlated.

(3) "Optimal" / "observed" wave forcing ->

• The control GW spectrum corresponds to the unique combination of wave flux and spectral width that yields the "observed" QBO amplitude (σ) and period (τ) according to (*).

 $[\sigma(25 \ km) - 33 \ m/s]^2$ $[\sigma(20 \ km) - 19 \ m/s]^2$ $[\tau(25 \ km) - 28 \ months]^2$ - (*) $[33 m/s]^2$ $[19 m/s]^2$ $[28 months]^2$



• The mean wave flux and spectral width required to capture the "observed" QBO in the 1D model are remarkably similar to those found in higher complexity models.

(5) Emulation

• We train different emulators using the "optimal" GW flux distribution for the training data.

$$-\frac{1}{\rho}\frac{\partial}{\partial z}\sum_{i}A_{i}\exp\left\{-\int_{z=z_{1}}^{z}\frac{\alpha(z')N}{k_{i}(u-c_{i})^{2}}dz'\right\} \rightarrow \text{Emulator}(u, F_{S0}, c_{w})$$



• Emulated solutions are stable for 108 years when ran "online" and are "faithful" to the control QBO period and amplitude.









(4) Control's sensitivity to GW spectrum

• The long-term QBO response is (unrealistically) regular, compared to both observations and higher complexity models. • Control run is stable for 108 years with a realistic QBO period.



• The dependencies of the amplitude and period on the GW wave flux and spectral width agree with both lower (HL72²) and P77³) and higher (G22⁴) complexity models.

(6) Generalization

 How well do emulators trained on a single source distribution generalize to nearby source distributions?



• Emulators <u>capture</u> the qualitative sensitivity of the QBO's <u>amplitude</u> to changes in the wave flux and spectral width. • Emulators <u>struggle</u> to capture the qualitative sensitivity of the QBO's period to changes in the wave flux and spectral width.







(7) Calibration

• Our emulators were trained on the optimal/observed source distribution, but a model's source distribution can be biased. Consider the emulated solution forced by the biased source distribution indicated by black * in box (3):



• How can we adjust the data-driven GW parameterization to yield the desired QBO statistics as in (*)?

• One approach is to re-map the biased source distribution to the optimal source distribution:



 $\{F_{S0}, c_w\} \rightarrow CDF_{Optimal}^{-1}(CDF_{Biased}(\{F_{S0}, c_w\}))$





¹Alexander, M. J., & Dunkerton, T. J. (1999). A spectral parameterization of mean-flow forcing due to breaking gravity waves. Journal of the Atmospheric Sciences, 56(24), 4167-4182.

²Holton, J. R., & Lindzen, R. S. (1972). An updated theory for the quasi-biennial cycle of the tropical stratosphere. Journal of Atmospheric Sciences, 29(6), 1076-1080.

³Plumb, R. A. (1977). The interaction of two internal waves with the mean flow: Implications for the theory of the quasi-biennial oscillation. Journal of Atmospheric Sciences, 34(12), 1847-1858.

⁴Garfinkel, C. I., Gerber, E. P., Shamir, O., Rao, J., Jucker, M., White, I., & Paldor, N. (2022). A QBO Cookbook: Sensitivity of the Quasi-Biennial Oscillation to Resolution, Resolved Waves, and Parameterized Gravity Waves. Journal of Advances in Modeling Earth Systems, 14(3), e2021MS002568.