

# Estimating the Meridional Extent of Adiabatic Mixing in the Stratosphere using Age-of-Air

Aman Gupta<sup>1,2,3</sup>, Marianna Linz<sup>4</sup>, Jezabel Curbelo<sup>5</sup>, Olivier Pauluis<sup>2</sup>, Edwin  
P. Gerber<sup>2</sup>, Douglas E. Kinnison<sup>6</sup>

<sup>1</sup>Meteorological Institute Munich, Ludwig-Maximilian University, Munich, BY, Germany

<sup>2</sup>Center for Atmosphere-Ocean Science, Courant Institute of Mathematical Sciences, New York, New  
York, USA

<sup>3</sup>Department of Earth System Science, Stanford University, Stanford, CA, USA

<sup>4</sup>Department of Earth and Planetary Sciences and School of Engineering and Applied Sciences, Harvard  
University, Cambridge, MA, USA

<sup>5</sup>Departament de Matemàtiques, Universitat Politècnica de Catalunya, Barcelona, Spain

<sup>6</sup>Atmospheric Chemistry Observations and Modeling Laboratory, National Center for Atmospheric  
Research, Boulder, CO, USA

## Key Points:

- The isentropic formulation of the leaky pipe stratospheric transport model (Linz et al., 2021) is used to estimate midlatitude mixing fluxes
- A new metric, which quantifies the meridional range of air parcels being mixed across transport barriers, is proposed to estimate mixing
- The deep tropical stratosphere is remarkably isolated and mixes with the extra-tropics only in the uppermost stratosphere

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Corresponding author: Aman Gupta, [Aman.Gupta@lmu.de](mailto:Aman.Gupta@lmu.de)

**Abstract**

Wave-induced adiabatic mixing in the winter midlatitudes is one of the key processes impacting stratospheric transport. Understanding its strength and structure is vital to understanding the distribution of trace gases and their modulation under a changing climate. age-of-air is often used to understand stratospheric transport, and this study proposes refinements to the vertical age gradient theory of Linz et al. (2021). The theory assumes exchange of air between a well-mixed tropics and a well-mixed extratropics, separated by a transport barrier, quantifying the adiabatic mixing flux across the interface using age-based measures. These assumptions are re-evaluated and a refined framework that includes the effects of meridional tracer gradients is established to quantify the mixing flux. This is achieved, in part, by computing a circulation streamfunction in age-potential temperature coordinates to generate a complete distribution of parcel ages being mixed in the midlatitudes. The streamfunction quantifies the “true” age of parcels mixed between the tropics and the extratropics. Applying the revised theory to an idealized and a comprehensive climate model reveals that ignoring the meridional gradients in age leads to an underestimation of the wave-driven mixing flux. Stronger, and qualitatively similar fluxes are obtained in both models, especially in the lower-to-middle stratosphere. While the meridional span of adiabatic mixing in the two models exhibits some differences, they show that the deep tropical pipe, i.e. latitudes equatorward of  $15^\circ$  barely mix with older midlatitude air. The novel age-potential temperature circulation can be used to quantify additional aspects of stratospheric transport.

**1 Introduction**

The large-scale stratospheric circulation, known as the Brewer-Dobson Circulation (BDC), plays a primary role in transporting long-lived trace gases throughout the stratosphere, thereby determining their spatial distributions. The BDC brings tropospheric air up into the stratosphere through the tropical tropopause and transports the air vertically and poleward. The breaking of planetary waves in the upper stratosphere and synoptic waves in the lower stratosphere both drives this meridional circulation across isentropes (diabatic) and mixes air horizontally along isentropic surfaces (adiabatic). This adiabatic mixing moves tracers over large spatial scales and plays an important role in exchanging midlatitude air with tropical air (Plumb, 2002). From a Lagrangian perspective, an air parcel entering the stratosphere will experience both diabatic advection and

53 adiabatic mixing over the course of its time in the stratosphere (Hall & Plumb, 1994;  
54 Garny & Randel, 2016).

55 The trace gases we observe in the stratosphere tend to be those that are long-lived.  
56 On short timescales, these gases can be treated as passive tracers advected by the back-  
57 ground flow. When the relative importance of the chemistry is small, the trace gas con-  
58 centration is dominantly determined by transport. It has been found that isopleths (sur-  
59 faces of constant mixing ratio) of different trace gases with varied chemistry have nearly  
60 identical shapes (Mahlman et al., 1986; Plumb & Ko, 1992). For instance, trace gases  
61 exhibit weak meridional gradients in the midlatitudes due to rapid horizontal mixing.  
62 In contrast, observations of trace gases reveal sharp local gradients in concentration in  
63 the subtropics and near the edge of the polar vortex, indicating existence of transport  
64 barriers (Neu et al., 2003; Shah et al., 2020).

65 Understanding the mixing across these transport barriers is important for climate  
66 and climate change: the distribution of trace gases determines their associated radiative  
67 forcing, and mixing plays a key role in transporting ozone-depleting substances into the  
68 polar vortex where they act as catalysts for ozone depletion (Lee et al., 2001). Intensi-  
69 fication in isentropic mixing has been known to increase the residence time of trace gases  
70 in the stratosphere (Neu & Plumb, 1999; Garny et al., 2014), increasing their likelihood  
71 of participating in ozone-destroying chemical reactions. Isentropic mixing is strongly in-  
72 fluenced by the mean flow and the wave-propagation conditions, especially in the win-  
73 ter stratosphere (Charney & Drazin, 1961; Holton et al., 1995). Moreover, changes in  
74 mixing can further project on to changes in the BDC strength itself, as enhanced mix-  
75 ing can potentially increase the diabatic upwelling of mass and trace gases into the strato-  
76 sphere. Therefore, these non-linear changes will collectively play a key role in determin-  
77 ing transport trends throughout the stratosphere in a changing climate (Ploeger et al.,  
78 2012, 2015; Abalos et al., 2016).

79 In order to understand short-term changes and long-term trends in isentropic mix-  
80 ing in the stratosphere, it is essential to first accurately estimate the wave-induced mix-  
81 ing fluxes. Several past studies have attempted to investigate the spatial structure of mix-  
82 ing through use of both dynamical and tracer-based metrics (Konopka et al., 2009; Aba-  
83 los & de la Cámara, 2020). For instance, eddy diffusivity, which depends on the eddy  
84 flux of potential vorticity and vorticity gradients, has been used to quantify mixing ex-

clusively using dynamical quantities (Plumb & Mahlman, 1987; Schneider, 2004). This idea has also been extended to measure eddy diffusivity of a tracer by redefining the diffusivity in terms of eddy flux and meridional gradient of the trace gas (Nakamura, 2001). Many other studies have analyzed the seasonality and long-term trends in adiabatic mixing through use of effective diffusivity (Allen & Nakamura, 2001; Shuckburgh et al., 2001; Chen & Plumb, 2014; Abalos et al., 2016), or other methods including Lyapunov diffusivity (Shuckburgh et al., 2009) and Lagrangian diffusivity (Curbelo et al., 2021).

While each method offers physical insights, each has its own set of limitations. For instance, studies using effective diffusivity frequently employ a coordinate transformation and work in the “equivalent latitude” space, instead of regular latitudes. This makes it non-trivial to connect changes and trends in mixing in the specialized coordinates to changes in true mixing; especially as the equivalent latitude is itself defined in terms of tracer contours which may substantially change over time. Moreover, computing effective diffusivity requires computing multiple spatial derivatives and integrals, which is not always readily possible for limited observational data.

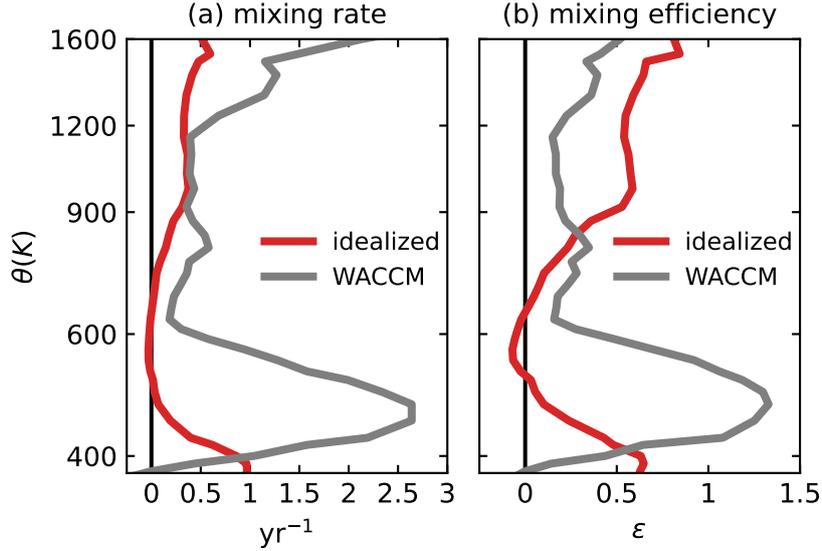
An alternative approach is to formulate the transport-dynamics coupling using theoretical or simple models of stratospheric transport (Plumb, 1996; Neu & Plumb, 1999; Ray et al., 2016). In order to mathematically model trace gas transport, these models study advection of an idealized tracer called “age-of-air” (Hall & Plumb, 1994; Waugh & Hall, 2002). Age-of-air quantifies how long an air parcel has been in the stratosphere and can be defined analytically. It is an effective tool to both quantify the transport timescales of trace gases in the stratosphere, and to study the transport of observed trace gases (Hall & Plumb, 1994). This is because, like many trace gases, one formulation of the age-of-air tracer has no spatial sources/sinks in the stratosphere (Boering et al., 1996); only a linear trend at the domain boundary. Thus, there exists a compact relation between the idealized tracer age-of-air and other chemically active trace gases (Plumb & Ko, 1992; Plumb, 2007). In fact, it is possible to directly estimate age-of-air from satellite observations of various atmospheric trace gases (Waugh & Hall, 2002; Linz et al., 2017), and age can also be readily computed in climate models either through time-lag analysis or through use of an idealized “clock” tracer (Hall et al., 1999; Garcia et al., 2011; Gerber, 2012). A similar approach has been used by several past studies focused on using the age-of-air distribution to infer dynamical properties of the stratosphere (Ray et al., 2010; Garny et al., 2014; Linz et al., 2016, 2021).

118           Of particular interest to this study are the recent works of Linz et al. (2016) and  
119 Linz et al. (2021), which proposed a theoretical framework using age-of-air to estimate  
120 the magnitude and vertical structure of both the diabatic circulation strength and the  
121 midlatitude mixing flux in the stratosphere. They used age-of-air to assess transport in  
122 the stratosphere and connect it to the large-scale circulation. Simply put, Linz et al. (2016)  
123 re-formulated the tropical “leaky pipe” model of Neu and Plumb (1999) in isentropic co-  
124 ordinates and established an inverse relationship between the diabatic circulation strength  
125 and the difference between tropical and extratropical age-of-air. Further, Linz et al. (2021)  
126 proposed a vertical age gradient theory which connects the net aging of air in the trop-  
127 ical “pipe” to the combined effects of aging due to the slow diabatic advection within  
128 the tropics and to the adiabatic mixing of air between the tropical pipe and the extra-  
129 tropics.

130           Both the age-of-air and the transport diagnostics proposed in the studies can be  
131 computed using satellite observations and/or climate models through straightforward  
132 integration. Therefore, this approach can be potentially used to directly and quantita-  
133 tively connect observations to theory and models. This enables inference of dynamical  
134 properties of the stratosphere that are difficult to directly measure using satellite mea-  
135 surements of trace gases, as explore by Linz et al. (2017).

136           In this study, we propose a framework to improve the mixing flux estimates obtained  
137 by Linz et al. (2021). To motivate this analysis, estimates of mixing rates derived by ap-  
138 plying their theory for two different models, one idealized (GFDL-FV3) and one com-  
139 prehensive (WACCM), are shown in Figure 1(a). Qualitatively similar mixing structures  
140 are obtained for the two models in that the mixing maximizes in the lower stratosphere:  
141 near 400 K for the idealized model (red) and near 500 K for the comprehensive model  
142 (gray). Above this level, the mixing rapidly decreases to near-zero in the middle strato-  
143 sphere, before exhibiting a small increase again in the upper stratosphere. The level of  
144 maximum mixing differs for the two models on account of a lower tropopause in the ide-  
145 alized model (Linz et al., 2021).

146           The models, however, show some key differences in the vertical mixing structure  
147 as well, observed in both the mixing rates (Figure 1(a)) and mixing efficiency (Figure  
148 1(b)). Mixing efficiency, a dimensionless quantity, is a ratio of the mixing flux to the net  
149 poleward mass flux and captures the vertical mixing structure in the stratosphere. Apart



**Figure 1.** (a) Derived mixing rate of air (in  $\text{yr}^{-1}$ ) from the winter midlatitudes stratosphere into the tropical stratosphere inferred from (red) an idealized finite volume dynamical core (FV3) with a perpetual January climatology and (grey) a comprehensive model (WACCM) for NDJFM months. Linz et al. (2021) define the mixing rate as the ratio of the estimated mixing flux (units  $\text{kg}/\text{K}/\text{s}$ ) and the horizontally integrated isentropic density (units  $\text{kg}/\text{K}$ ). (b) Mixing efficiency (a proxy for mixing) derived from the inferred mixing rates in (a). Mixing efficiency ( $\epsilon$ ) is a dimensionless quantity and is defined as the ratio of the net mixing flux and the net poleward flux. The net poleward flux,  $\mu_{net}$ , is defined as the vertical convergence of the net diabatic flux in the tropical pipe. For exact definitions see Equation (17) of Linz et al. (2021).

150 from the differences in mixing peaks between the idealized (400 K) and the comprehen-  
 151 sive model (500 K), the idealized model exhibits a significantly weaker mixing over the  
 152 400-600 K interval. These differences cannot be attributed to differences in the tropopause  
 153 height alone. In fact, the mixing is weak to the extent that the estimated mixing flux  
 154 and efficiency, which should be non-negative by definition, assumes non-sensical nega-  
 155 tive values around 600 K in the idealized model. Similar mixing differences and nega-  
 156 tive mixing estimates were also found among the models in the annual mean calculations  
 157 of Linz et al. (2021) (their Figure 5). More pronounced negative mixing fluxes were ob-  
 158 tained by Gupta et al. (2021), who used the theory to compare mixing fluxes among a  
 159 broad range of idealized models.

160 Rectifying the false negative flux estimates, reconciling the difference in mixing be-  
 161 tween the two models, and ultimately obtaining more accurate mixing fluxes, are the goals  
 162 of this study. We show that “negative” mixing appears due to an incorrect assumption  
 163 of a perfectly-mixed tropics and midlatitudes. The Linz et al. (2021) theory assumes fast  
 164 horizontal mixing within each of the tropical and extratropical “pipes”, so neglecting tracer  
 165 gradients within each region. This effectively implies the deep tropics are mixing just  
 166 as much as with the extratropics as air at the edge of the pipe, and that the polar air  
 167 is mixing just as much with the tropics as midlatitude air. This incorrect assumption  
 168 leads to a potential underestimation of the mixing fluxes across the mixing barrier. In  
 169 reality, adiabatic mixing is more localized to the winter midlatitudes with only occasional  
 170 intrusions into the polar vortex or subtropics.

171 It is possible to get a more accurate estimate of mixing fluxes by accounting for  
 172 subtropical gradients. We refine the theory of Linz et al. (2021) accordingly to account  
 173 for the sensitivity of the mixing estimates to strong subtropical gradients. These refine-  
 174 ments enable us to improve the mixing flux estimates without compromising the sim-  
 175 plicity of their model. Both the original mixing theory and our refinement to correct for  
 176 the strong meridional gradients within each “pipe” are discussed in Section 2. Section  
 177 3 presents the model setup for the idealized and comprehensive model assessed in this  
 178 study. We apply the proposed theory and discuss our results in Section 4. Finally, we  
 179 draw conclusions from the analysis and discuss potential avenues for further research in  
 180 Section 5.

## 181 **2 A refined theory to estimating the mixing flux**

182 The isentropic formulation of Linz et al. (2016, 2021) begins with the horizontally  
 183 integrated, mass-flux weighted ages  $\Gamma_u$  and  $\Gamma_d$  over respective regions of diabatic upwelling  
 184 and diabatic downwelling respectively. They are defined as:

$$\Gamma_u(\theta) = \frac{\int_u \rho_\theta \dot{\theta} \Gamma dA}{\int_u \rho_\theta \dot{\theta} dA} ; \quad \Gamma_d(\theta) = \frac{\int_d \rho_\theta \dot{\theta} \Gamma dA}{\int_d \rho_\theta \dot{\theta} dA} \quad (1)$$

185 where  $\theta$  is the potential temperature,  $\dot{\theta}$  is the diabatic velocity,  $\rho_\theta = \frac{-1}{g} \left| \frac{d\theta}{dp} \right|$  is the isen-  
 186 tropic density,  $\Gamma$  is the age-of-air, the integrand  $dA$  is the infinitesimal area element in  
 187 the latitude-longitude space,  $p$  is the pressure, and the subscripts  $\int_u$  and  $\int_d$  respectively

188 denote selective integration over regions of diabatic upwelling ( $\dot{\theta} > 0$ ) and diabatic down-  
 189 welling ( $\dot{\theta} < 0$ ).

190 Linz et al. (2021) showed that for a steady circulation, in the limit of no vertical  
 191 diffusion and fast horizontal mixing, the vertical age gradient in the tropical pipe is the  
 192 sum of aging due to vertical advection and aging due to adiabatic mixing. Mathemat-  
 193 ically, this is expressed as :

$$\underbrace{\frac{d\Gamma_u}{d\theta}}_{\text{T1 : vertical gradient}} = \underbrace{\frac{\sigma_u}{\mathcal{M}}}_{\text{T2 : advection}} + \underbrace{\frac{\mu_{mix} \Delta\Gamma}{\mathcal{M}}}_{\text{T3 : bulk mixing}} \quad (2)$$

194 where  $\theta$  is the potential temperature,  $\Delta\Gamma = \Gamma_d - \Gamma_u$  is the age difference between the  
 195 extratropics and the tropics,  $\mathcal{M}$  is the diabatic mass flux,  $\sigma_u$  (units of kg/K) is the isen-  
 196 tropic density ( $\rho_\theta$ ) horizontally integrated over region of upwelling and  $\mu_{mix}$  is the mass  
 197 flux per Kelvin that mixes the midlatitude air with the tropical air. The advection term  
 198 T2 captures the net aging of air as it is advected by the diabatic circulation up the trop-  
 199 ical pipe: the mass of air per unit potential temperature (kg/K) divided by the flux of  
 200 mass (kg/s)—literally, how long it takes the air to rise one unit of potential tempera-  
 201 ture. The bulk mixing term T3 captures the total aging of the tropical pipe by older air  
 202 mixed in from the extratropics:  $\Delta\Gamma$  quantifies how much older this extratropical air is  
 203 relative to the tropics, and  $\mu_{mix}/\mathcal{M}$  quantifies the relative contribution of this air per  
 204 unit Kelvin.

205 All the terms in the mixing equation except  $\mu_{mix}$  can be computed from the winds  
 206 and age-of-air. Thus, the mixing flux  $\mu_{mix}$  is estimated as a residual in Equation 2. For  
 207 a more detailed discussion, see Section 3 of Linz et al. (2021) and Appendix A. As we  
 208 will show, computing the mixing as a residual allows errors in other terms to corrupt the  
 209 implied mixing flux.

## 210 **2.1 Modification to the vertical age gradient equation**

211 Both the original leaky pipe formulation (Neu & Plumb, 1999) and the isentropic  
 212 formulation of Linz et al. (2021) assume fast horizontal mixing of trace gases within the  
 213 upwelling and downwelling regions. With sufficiently fast mixing, the horizontal gradi-  
 214 ent of age-of-air within each region can be ignored, and thus the age of all parcels in each  
 215 region can be simply assumed to be  $\Gamma_u(\theta)$  and  $\Gamma_d(\theta)$ , the corresponding mass flux weighted  
 216 age-of-air in the upwelling/downwelling regions. It follows that the entrainment flux trans-

217 ports parcels with a fixed age  $\Gamma_u$  across the barrier towards higher latitudes, and the mix-  
 218 ing flux transports parcels with a fixed age  $\Gamma_d$  back to lower latitudes. The mixing term  
 219 T3 in Equation 2 is therefore interpreted as the bulk mixing flux  $\mu_{mix}$  which mixes the  
 220 meridional age difference  $\Delta\Gamma$  between the two regions.

221 The meridional profile of age-of-air estimated from observations and models, how-  
 222 ever, exhibits sharp tracer gradient around the sub-tropical barrier (Waugh & Hall, 2002).  
 223 For instance, age at 50 hPa from the benchmark tests of Gupta et al. (2020) (shown in  
 224 Figure 4(b) later) linearly increases from 3 Yr at the equator to 5.5 Yr around the sub-  
 225 tropical barrier. Further poleward, in the surf-zone, the meridional gradient is much weaker  
 226 on account of wave-induced mixing. Therefore, assuming a perfectly-mixed tropics leads  
 227 to an overestimation of the actual age difference between air being mixed between the  
 228 tropics and the extratropics. The midlatitude mixing, which is the main driver of hor-  
 229 izontal churning of trace gases, does not generally extend all the way to the equatorial  
 230 region. As will be shown in later sections, the deep tropics, especially in the middle and  
 231 lower stratosphere, are fairly isolated from the wave-induced mixing fluxes that originate  
 232 in the surf-zone. Consequently, assuming perfectly mixed tropics and extratropics leads  
 233 to an underestimation of the midlatitude mixing flux  $\mu_{mix}$  from Equation 2.

234 Consider perturbations  $\delta\Gamma_u$  and  $\delta\Gamma_d$  to the mean ages  $\Gamma_u$  and  $\Gamma_d$  respectively on  
 235 a given isentrope, and assume that instead of air with the tropical mean age  $\Gamma_u$  mixing  
 236 with air with the extratropical age  $\Gamma_d$ , slightly older air in the tropics with age  $\Gamma_u + \delta\Gamma_u$   
 237 mixes with a slightly younger air from the extratropics with age  $\Gamma_d - \delta\Gamma_d$ . Physically,  
 238 this captures the fact that the mixing is more localised around the turnaround latitude  
 239 and does not span the entirety of the two regions. Re-deriving the mixing equation with  
 240 this assumption yields the *revised mixing equation* :

$$\underbrace{\frac{d\Gamma_u}{d\theta}}_{\text{C1}} = \underbrace{\frac{\sigma_u}{\mathcal{M}}}_{\text{C2}} + \underbrace{\mu_{mix}^T \frac{\Delta\Gamma_{eff}}{\mathcal{M}}}_{\text{C3 : true mixing}} - \underbrace{\mu_{net} \frac{\delta\Gamma_u}{\mathcal{M}}}_{\text{C4 : enterainment freshening}} \quad (3)$$

241 where  $\Delta\Gamma_{eff} = \Delta\Gamma - (\delta\Gamma_u + \delta\Gamma_d)$  is the effective age difference being mixed across the  
 242 barrier, following the modification, and  $\mu_{net}$  is the net poleward flux defined as the ver-  
 243 tical diabatic flux convergence, i.e.  $\mu_{net} = -\frac{\partial\mathcal{M}}{\partial\theta}$ . Substituting  $\delta\Gamma_u = \delta\Gamma_d = 0$  yields  
 244 the original mixing equation of Linz et al. (2021). The detailed derivation is provided  
 245 in the Appendix. The advective term C2 on the right is identical to T2 in Equation 2.

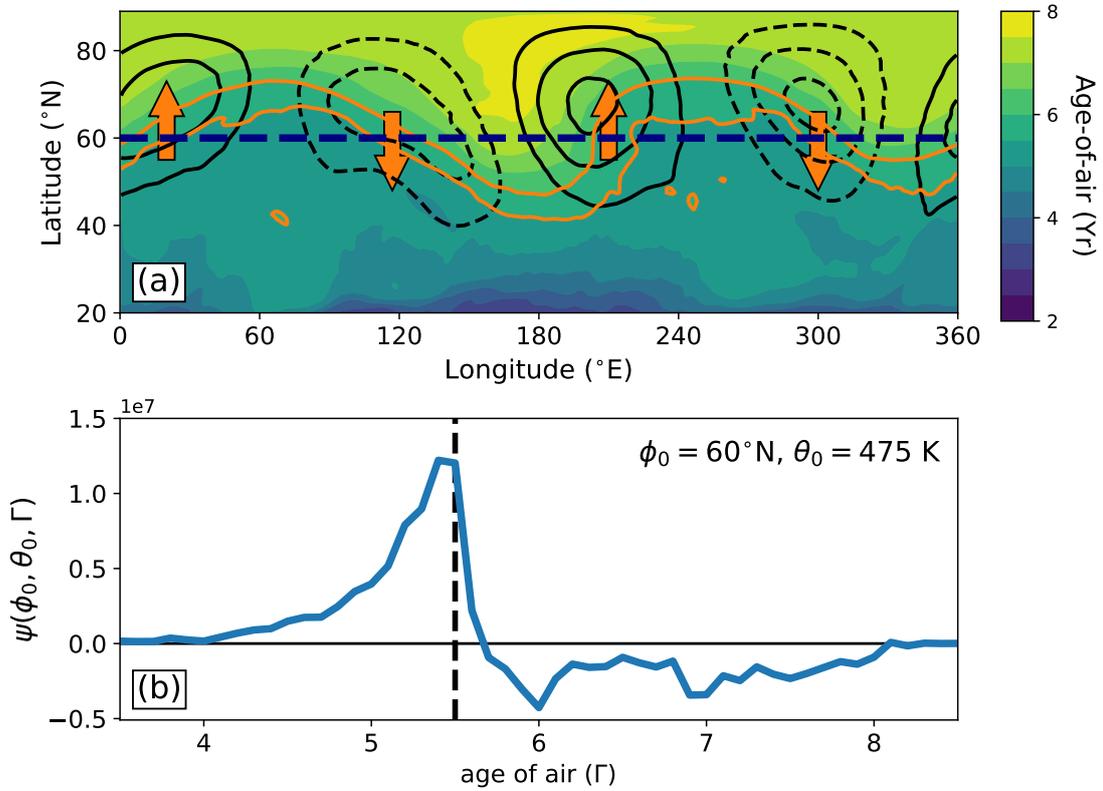
246 The total age difference  $\Delta\Gamma$  in term T3 in Equation 2 is now replaced with the effective  
 247 age difference  $\Delta\Gamma_{eff}$  in the true mixing term C3 in Equation 3. The age perturbations  
 248  $\delta\Gamma_u$  and  $\delta\Gamma_d$  should be non-negative, so  $\Delta\Gamma_{eff} \leq \Delta\Gamma$ ; the true age difference of the parcels  
 249 mixed between the two regions is smaller than the age difference  $\Delta\Gamma$ . In later sections,  
 250 it is shown that for the idealized dynamical core setup the perturbations  $\delta\Gamma_u$  and  $\delta\Gamma_d$   
 251 are indeed positive. This, however, cannot be said to be generally true for the compre-  
 252 hensive model WACCM due to complications in interpreting the annual cycle. An ad-  
 253 ditional term C4 now appears in Equation 3: the air leaving the tropics is older than the  
 254 mean age  $\Gamma_u$  and this term corrects the net poleward age transport. For non-negative  
 255 perturbations, C4 should also be non-negative and provide a positive offset/correction  
 256 to the mixing flux estimate from Equation 2. The updated C3 and new C4 terms pro-  
 257 vide a positive “correction” to yield the true mixing flux  $\mu_{mix}^T$ .

258 In order to compute the true mixing flux using equation 3, an estimate of the per-  
 259 turbation ages  $\delta\Gamma_u$  and  $\delta\Gamma_d$  is needed. To compute these, we define a  $\Gamma$ - $\theta$  circulation  
 260 distribution in the following subsection. The circulation distribution provides a compre-  
 261 hensive description of the age range of parcels being transported at a given latitude and  
 262 isentropic height allowing us to estimate the perturbation ages  $\delta\Gamma_u$  and  $\delta\Gamma_d$ . To differ-  
 263 entiate between the mixing estimates obtained from Equation 2 and 3, we will hereafter  
 264 refer to  $\mu_{mix}$  estimated from Equation 2 as the “bulk mixing flux” and  $\mu_{mix}^T$  estimated  
 265 from Equation 3 as the “true mixing flux”.

## 266 **2.2 Computing $\delta\Gamma_u$ and $\delta\Gamma_d$ with diabatic circulation as a function of** 267 **age**

268 The schematic in Figure 2(a) provides an intuitive introduction to the  $\Gamma$ - $\theta$  circu-  
 269 lation, showing the meridional wind (black) and age-of-air (color) on a fixed, arbitrar-  
 270 ily chosen, 475 K ( $\approx 70$  hPa) isentropic surface as a function of latitude and longitude.  
 271 Our goal is to switch from viewing this circulation in longitude to viewing this circula-  
 272 tion in age: at  $60^\circ\text{N}$ , for instance, what is the age of mass moving poleward and equa-  
 273 torward? We see that the poleward advected air across this latitude is, on average, younger  
 274 than the equatorward advected air. The  $\Gamma$ - $\theta$  circulation provides the entire distribution  
 275 of this transport.

276 To compute the  $\Gamma$ - $\theta$  circulation, we first isolate a chosen isentropic surface with  $\theta =$   
 277  $\theta_0$ . Further, we selectively consider only a chosen  $\Gamma = \Gamma_0$  contour on this chosen isen-  
 278 tropes. Finally, we compute the mean meridional mass transport exclusively associated  
 279 with the chosen potential temperature and age-of-air level. This provides the net mass  
 280 flux associated with the whole range of ages associated with the transported parcels on  
 281 the selected  $\theta_0$ -isentrope.



**Figure 2.** (a) A schematic providing physical intuition behind the mathematical formulation of the  $\Gamma$ - $\theta$  circulation defined in Equation 4. The solid and dashed curves show the positive and negative meridional velocity on the 475 K isentropic surface, as marked by the orange arrows. The colors show the contours of age-of-air which is transported by the meridional flow across a given latitude (dashed blue). The 5.5 yr age contour is highlighted with orange borders. (b)  $\Gamma$ - $\theta$  circulation at  $60^{\circ}$ N on the 475 K isentrope, as a function of age. The vertical bar in dashed black marks an age of 5.5 Yr corresponding to the orange contours in (a). The age  $\Gamma$  in the horizontal axis corresponds to the integrand  $\Gamma'$  in Equation 5.

282 The Dirac-delta formulation of the diabatic streamfunction developed by Pauluis  
 283 et al. (2009), computes meridional mass transport in moist isentropic coordinates in the  
 284 troposphere through selective binning of mass transport into potential temperature bins  
 285 or levels. Pauluis et al. (2009) use the approach to estimate a joint distribution of mass  
 286 transport as a function of both dry and moist potential temperature by using two Dirac-  
 287 delta functions (see Equation (1) of Pauluis et al. (2009) for details). This study follows  
 288 a similar approach to obtain the joint distribution of the meridional mass transport as  
 289 a function of both potential temperature ( $\theta$ ) and age-of-air ( $\Gamma$ ). Mathematically, this is  
 290 akin to using two Dirac-delta functions, one for potential temperature and one for age  
 291 in order to bin the meridional mass transport according to both the potential temper-  
 292 ature and the mean age of the parcels being transported. The joint  $\Gamma$ - $\theta$  meridional mass  
 293 transport distribution,  $\psi$ , can thus be expressed as :

$$\psi(\phi, \theta_0, \Gamma_0) = \frac{2\pi R \cos \phi}{g} \int_0^{2\pi} \int_0^{p_s} v \delta(\theta - \theta_0) \delta(\Gamma - \Gamma_0) dp d\lambda \quad (4)$$

294 where  $\phi$  is the latitude,  $\theta_0$  and  $\Gamma_0$  are the select potential temperature and age bins at  
 295 which the circulation is sampled,  $v$  is the meridional velocity on model pressure levels,  
 296  $\delta(\cdot)$  is the Dirac-delta function,  $p_s$  is the surface pressure,  $dp$  and  $d\lambda$  represent vertical  
 297 and zonal integration respectively,  $R$  is the radius of earth, and  $g$  is the acceleration due  
 298 to gravity. To compute the zonal averages in the integral on isentropic surfaces, the isen-  
 299 tropic binning technique introduced in Yamada and Pauluis (2015) was used, as it al-  
 300 lows us to avoid direct computation of the isentropic density. For brevity, we hereafter  
 301 refer to the quantity  $\psi$  as the  $\Gamma$ - $\theta$  *circulation*. In the northern hemisphere, positive val-  
 302 ues of  $\psi$  indicate poleward transport and negative values, equatorward transport. The  
 303 signs are reversed for the southern hemisphere.

304 Revisiting Figure 2(a), for data originally on pressure levels, the Dirac-delta func-  
 305 tion  $\delta(\theta - \theta_0)$  isolates the isentropic surface with  $\theta = \theta_0$ . Moreover, the Dirac-delta  
 306 function  $\delta(\Gamma - \Gamma_0)$  selectively considers only the  $\Gamma = \Gamma_0$  contour on the chosen isen-  
 307 tropic surface. For  $\Gamma_0 = 5.5$  Yr, for instance, the two delta functions isolate the 5.5 Yr  
 308 age contour, highlighted in orange, allowing us to compute the meridional mass trans-  
 309 port along that contour. Finally, computing zonal mean along a latitude circle captures  
 310 the net mean meridional circulation i.e., computing the net meridional transport only  
 311 in the regions where the orange highlighted contours meets the (say) 60°N latitude cir-  
 312 cle (dashed blue). We compute the mass transport for a wide range of age levels from  
 313 0 to 15 Yr and show the mass streamfunction on 475 K and 60°N latitude in Figure 2(b).

314 For a given latitude circle, such as that shown in dashed blue, the net mass transport  
 315 is distributed over a broad range of parcel ages. In this case, the circulation moves air  
 316 parcels with age 4 Yr to 5.7 Yr poleward and air parcels with age 5.7 Yr to 8 Yr equa-  
 317 torward. The equatorward transport is spread over a much wider range at 60°N due to  
 318 sharp tracer gradients poleward of 60°N on account of the polar vortex edge barrier. This  
 319 can be clearly seen in Figure 2(a).

320 For a chosen latitude and potential temperature,  $\psi$  provides a distribution of merid-  
 321 ional mass transport as a function of age. Therefore, it enables estimation of the com-  
 322 plete range of ages of the parcels being transported by the meridional circulation of the  
 323 stratosphere across a given latitude and on a given isentrope. This distribution is used  
 324 to compute the true age being mixed around the transport barrier for a given isentrope,  
 325 which in turn allows us to compute  $\delta\Gamma_u$  and  $\delta\Gamma_d$  needed for the refined theory in Equa-  
 326 tion 3.

327 To estimate the true age of the parcels being mixed between the upwelling and down-  
 328 welling regions and hence  $\delta\Gamma_u$  and  $\delta\Gamma_d$ , we calculate the  $\Gamma$ - $\theta$  circulation-weighted ages  
 329 separately over the intervals of poleward vs equatorward mass transport at the turnaround  
 330 latitude. This yields the *effective ages*  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$  of the parcels being mixed across  
 331 the turnaround latitude on this potential temperature surface. For latitudes in the north-  
 332 ern hemisphere, the ages, expressed as a circulation-weighted average, can be expressed  
 333 as:

$$\Gamma_{u,eff} = \frac{\int_{\psi>0} \psi \Gamma' d\Gamma'}{\int_{\psi>0} \psi d\Gamma'} ; \Gamma_{d,eff} = \frac{\int_{\psi<0} \psi \Gamma' d\Gamma'}{\int_{\psi<0} \psi d\Gamma'} \quad (5)$$

334 where the variable of integration,  $\Gamma'$ , is the age-of-air, and the integration limit  $\psi > 0$   
 335 means integration only over positive values of  $\psi$ . In our computations,  $\Gamma'$  ranges from  
 336 0 Yr to 15 Yr, which is the maximum age-of-air in our model runs. The difference be-  
 337 tween the effective and the mass flux-weighted ages is defined as the perturbation ages  
 338  $\delta\Gamma_u$  and  $\delta\Gamma_d$ . More precisely, the  $\Gamma$ - $\theta$  circulation is used to estimate  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$ ,  
 339 and the perturbations are calculated as  $\delta\Gamma_u = \Gamma_{u,eff} - \Gamma_u$  and  $\delta\Gamma_d = \Gamma_d - \Gamma_{d,eff}$ .

### 340 **2.3 Eddy Diffusivity**

341 The isentropic eddy diffusivity is an alternative method to assess the meridional  
 342 profile of the mixing flux of age. Plumb and Mahlman (1987) and Schneider (2004) de-  
 343 fine eddy diffusivity as a diffusive parameterization of eddy fluxes in terms of zonal mean

344 quantities to obtain closure of the tracer continuity equation. Nakamura (1996), Shuckburgh  
 345 et al. (2001), and Abalos et al. (2016) quantify mixing using an effective diffusivity based  
 346 on a Lagrangian treatment of mixing and transformation of equations based on tracer-  
 347 area coordinates. For this study, the former approach is followed on account of its di-  
 348 rectness. The isentropic eddy diffusivity is defined as the ratio of the net eddy transport  
 349 of age to the mean meridional age gradient. The eddy age flux  $F_{eddy}$  is computed as the  
 350 difference of total meridional age flux and the mean meridional advection of age by the  
 351 mean flow. Mathematically, in isentropic coordinates, it is expressed as:

$$F_{eddy}(\phi, \theta) = \overline{\rho_\theta v \Gamma} - (\overline{\rho_\theta v}) \tilde{\Gamma} \quad (6)$$

352 where,  $\tilde{\Gamma}(\phi, \theta) = \overline{\rho_\theta \Gamma} / \overline{\rho_\theta}$  is the mass weighted age in isentropic coordinates,  $\rho_\theta$  is the  
 353 isentropic density,  $v$  is the meridional velocity, and overbar denotes zonal averaging on  
 354 fixed isentropes. The isentropic eddy diffusivity  $\mathcal{D}_{eff}$  (units  $\text{m}^2 \text{s}^{-1}$ ) is then defined as  
 355 the ratio of the density-scaled eddy flux and the meridional gradient of age as :

$$\mathcal{D}_{eff} = \frac{1}{\overline{\rho_\theta}} \frac{-F_{eddy}}{\partial_y \tilde{\Gamma}} \quad (7)$$

356 where  $\partial_y = (1/R)\partial_\phi$  is the meridional gradient of the mean isentropic age. The eddy  
 357 diffusivity is expected to peak in the stratospheric midlatitudes on account of strong plan-  
 358 etary wave-induced eddy transport and weak meridional gradients in the surf zone.  $\mathcal{D}_{eff}$   
 359 can therefore be used to qualitatively compare the structure of midlatitude eddy mix-  
 360 ing.

### 361 **3 Models**

362 The revised mixing equation is tested using the two models introduced in Figure  
 363 1. The first is an idealized model with a prescribed equilibrium temperature profile, a  
 364 simple treatment of gravity wave drag as a Rayleigh drag at the model top, and no other  
 365 physical parameterizations in the stratosphere. The second is a comprehensive climate  
 366 model with a detailed representation of chemistry and radiation. The models have sim-  
 367 ilar numerics in that both employ a finite volume fluid dynamical solver, albeit on dif-  
 368 ferent horizontal grids. This eliminates a key source of uncertainty that can arise due  
 369 to differences in model numerics itself (Gupta et al., 2020). Working with a comprehen-  
 370 sive model and an idealized model also allows us to test how strongly the negative mix-

ing fluxes estimated using the vertical age gradient theory are related to complex parameterized processes.

### 3.1 Idealized Model: FV3

The study uses the Free Running model setup used in Gupta et al. (2020), with a finite volume dynamical core based on a cubed sphere grid. The core was developed at the Geophysical Fluid Dynamics Laboratory (GFDL) and is referred to as FV3 for short. The core employs finite volume schemes in both the vertical and the horizontal to solve the primitive equations (Putman & Lin, 2007). FV3 was built as the core of GFDL's Atmospheric Model, Version 3, AM3 (Donner et al., 2011), and a related non-hydrostatic version was recently adopted as the core of the National Center for Environmental Prediction Global Forecasting System. Further details on FV3 are provided in Gupta et al. (2020, Section 3 and Appendix A). Briefly, the model is driven with thermal forcings (Held & Suarez, 1994; Polvani & Kushner, 2002) detailed in Section 4.1 of Gupta et al. (2020). Newtonian relaxation to an analytically defined temperature state, which can be interpreted as a state of radiative-convective equilibrium, generates a perpetual northern-winter climatology. To quantify transport, a tracer with a concentration that is linearly increasing in time (a clock tracer) is introduced near the surface ( $p \geq 700$  hPa) as detailed in Gupta et al. (2020), Section 4.1. The clock tracer is used to compute the age-of-air, which provides a measure of transport timescales of trace gases in the stratosphere.

As in Gupta et al. (2020), the model was integrated for 10,000 days and the last 3,300 days were used for transport analysis. The model was integrated at a  $1^\circ \times 1^\circ$  horizontal resolution using a C90 horizontal grid with 80  $\sigma$ -p hybrid levels in the vertical. The levels were pure- $\sigma$  (terrain following) below 500 hPa, pure-pressure above 200 hPa and a linear combination of the two between 200 hPa and 500 hPa.

### 3.2 Comprehensive Climate Model: WACCM

The comprehensive climate model employed in the study is the same as that employed in Linz et al. (2021): the Community Earth System Model 1 Whole Atmosphere Community Climate Model (WACCM), an interactive chemistry-climate model (Garcia et al., 2017; Marsh et al., 2013) developed at the National Center for Atmospheric Research (NCAR). This model uses physical parameterizations to represent complex earth

401 system processes including atmospheric chemistry and radiation, and is based on a finite-  
 402 volume dynamical core (Lin, 2004) from the Community Atmosphere Model, version 4  
 403 (Neale et al., 2013). Its domain extends from the surface to 140 km, with 31 pressure  
 404 levels from 193 to 0.3 hPa. The horizontal resolution is  $2.5^\circ$  longitude  $\times$   $1.875^\circ$  latitude,  
 405 corresponding to the F19 horizontal grid.

406 The WACCM simulations are based on the Chemistry Climate Model Initiative REF-  
 407 C1 scenario (Morgenstern et al., 2017). An ideal clock tracer, to compute the age-of-air,  
 408 is included and is specified as in Garcia et al. (2011), with a uniform mixing ratio at the  
 409 lower boundary that is linearly increasing in time. The model is forced with observed  
 410 sea surface temperatures. The Quasi-Biennial Oscillation is nudged to observed winds,  
 411 but otherwise the model evolves freely. The model was integrated from 1979 to 2014. Fur-  
 412 ther details on WACCM are provided in Section 3 of Linz et al. (2021).

## 413 **4 Results**

414 The idealized FV3 model best allows us to contrast the original and revised mix-  
 415 ing equation. Because it is in perpetual northern hemisphere winter, there is a contin-  
 416 uous, strong mixing barrier associated with the Northern Hemisphere polar vortex. We  
 417 therefore expect the assumptions of Linz et al. (2021) to be less appropriate than in a  
 418 seasonally-varying context, where the annual breakdown of the vortex partially erodes  
 419 the barrier. This experiment provides a more stringent test of our revised mixing for-  
 420 mulation. Following that, we discuss our findings in the context of the comprehensive  
 421 WACCM model.

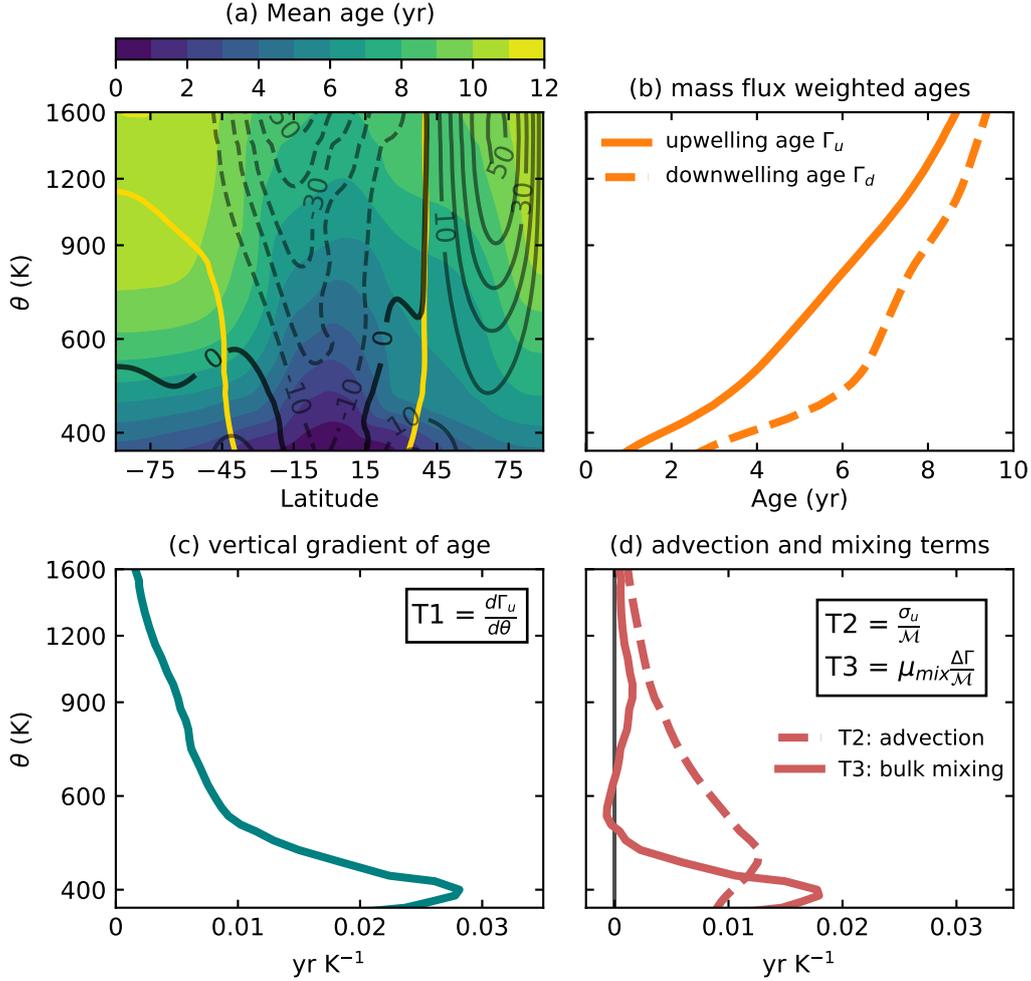
### 422 **4.1 Estimating the mixing fluxes using the refined theory**

423 The climatological winds generated in response to the prescribed diabatic heating  
 424 in FV3 comprises a strong polar vortex in the northern high latitudes and a strong east-  
 425 erly jet in the subtropics, Figure 3(a). The solid blue lines trace the turnaround latitude  
 426 in each hemisphere, i.e. the latitude associated with zero diabatic velocity. The “trop-  
 427 ical pipe” enclosed between the two blue curves is characterized by slow diabatic ascent  
 428 of mass. Likewise, the region poleward of the blue curves is characterized by slow dia-  
 429 batic descent of mass. We refer to these two partitions as upwelling and downwelling re-  
 430 gions respectively.

431 The climatological mean age-of-air profile (in color), obtained as a 10-year aver-  
 432 age, is well-stratified, monotonically increasing with height. At any given height, the youngest  
 433 air is found within the tropics, and the oldest air either in the winter or summer polar  
 434 region, depending on the level. The concave, vertically stacked contours in the equato-  
 435 rial region are consistent with slow vertical transport in the region; fresh air from the  
 436 tropical tropopause is vertically advected upward. Away from the equator, the age in-  
 437 creases monotonically and exhibits a strong gradient around the subtropical barrier. Pole-  
 438 ward of the barrier, in the surf-zone, an abrupt flattening of the age contours is created  
 439 by enhanced midlatitude mixing. In the winter hemisphere most of this mixing is induced  
 440 by breaking planetary waves, while in the summer hemisphere it is induced by a com-  
 441 bination of both synoptic-scale wave breaking in the lower stratosphere and slow diffu-  
 442 sive transport elsewhere. In winter high latitudes, a steep increase in potential vortic-  
 443 ity with latitude inhibits transport into and out of the polar vortex. Consequently, an  
 444 abrupt increase in age is observed near the edge of the polar vortex.

445 The age-of-air in Figure 3(a) when weighted by the diabatic mass flux and aver-  
 446 aged over the upwelling and downwelling regions, respectively, yields  $\Gamma_u(\theta)$  and  $\Gamma_d(\theta)$   
 447 (defined in equation 1), shown in Figure 3(b). The downwelling/midlatitude age,  $\Gamma_d$ , is  
 448 older than the upwelling/tropical age,  $\Gamma_u$ , throughout the stratosphere, with the differ-  
 449 ence  $\Delta\Gamma = \Gamma_d - \Gamma_u$  maximizing near 500 K in the lower stratosphere.

450 The mixing equation, Equation 2, connects the net vertical aging of air in the trop-  
 451 ical pipe to the aging by diabatic advection vs adiabatic mixing. The net aging, i.e., the  
 452 vertical gradient of the upwelling age (T1), is maximum near 400 K, above which it mono-  
 453 tonically decreases with height, Figure 3(c). In the lower stratosphere, a major fraction  
 454 of the aging can be attributed to mixing in the lower stratosphere, as shown in Figure  
 455 3(d) (solid curve). The other factor contributing to the aging is the reduction of verti-  
 456 cal motion on account of increased static stability as compared to the troposphere. At  
 457 400 K, the bulk mixing term (T3) accounts for an aging rate of  $0.018 \text{ yr K}^{-1}$  out of the  
 458 net vertical aging (T1) of  $0.028 \text{ yr K}^{-1}$ , Figure 3(c), as the synoptic-scale mixing dom-  
 459 inates in both the hemispheres. This fraction, however, rapidly decreases with height and  
 460 between 550-600 K almost all the aging is accounted for by the advective term (T2). In  
 461 fact, our computations yield slightly negative values of the bulk mixing term in this re-  
 462 gion, a clearly nonsensical result. In the upper stratosphere, the increase in the relative



**Figure 3.** (a) The climatological mean age profile (in color) and zonal mean zonal wind (in black), along with the zero diabatic velocity curve (in yellow) for the FV3 idealized model. (b-d) Individual terms of Equation 2 applied to the idealized model: (b) The diabatic flux weighted upwelling and downwelling ages,  $\Gamma_u$  (solid) and  $\Gamma_d$  (dashed), (c) the vertical gradient of upwelling age,  $\partial\Gamma_u/\partial\theta$ , i.e., term T1 in the mixing equation, and (d) advection (dashed) and bulk mixing (solid) terms i.e., terms T2 and T3 in the equation respectively. The term T3 is computed as the difference T1-T2, leading to spurious, negative values for the bulk mixing flux  $\mu_{mix}$ .

463 contribution by the bulk mixing term is due to enhanced mixing due to breaking plan-  
 464 etary waves.

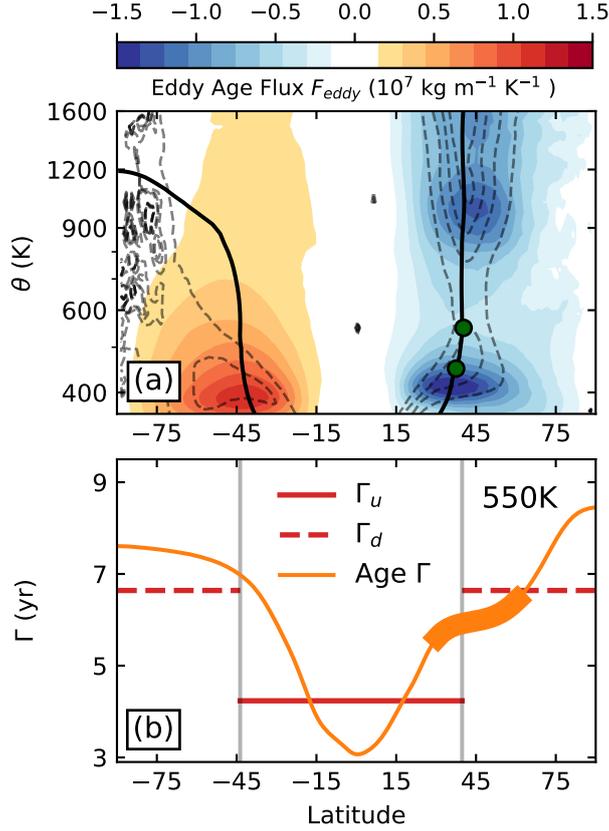
465 As was pointed out in Linz et al. (2021) and also shown in Figure 1, minor neg-  
 466 ative fluxes are obtained in the middle stratosphere both for the comprehensive WACCM

467 model (annual average) and for the idealized FV3 model. This “negative mixing” is not  
 468 a numerical artifact but rather a consequence of the assumption of fast horizontal mix-  
 469 ing within both the upwelling and downwelling regions. While this assumption allows  
 470 convenient assessment of age in the two regions using characteristic values  $\Gamma_u$  and  $\Gamma_d$ ,  
 471 it ignores the meridional gradient in age around the subtropical transport barrier. The  
 472 flux of older midlatitude air into the tropics is localized around the barrier itself, barely  
 473 churning the air in the deep tropics.

474 The bulk mixing term (T3) and bulk mixing flux  $\mu_{mix}$  in Equation 2 are computed  
 475 as residual terms in the mixing equation. To first verify that the mixing is always pos-  
 476 itive, we compute the meridional profile of the eddy flux of age,  $F_{eddy}$ , was computed and  
 477 is shown in Figure 4(a) (in color).  $F_{eddy}$  is computed from the eddy covariance in isen-  
 478 tropic coordinates, as in Equation 6. Negative and positive fluxes in the winter and sum-  
 479 mer hemisphere respectively indicate an equatorward transport of age by midlatitude  
 480 eddies. In the winter hemisphere, the eddy age flux has a two-peaked structure in the  
 481 vertical, with the first maximum at 425 K in the lower stratosphere (due to synoptic-  
 482 scale mixing) and the second maximum at around 1000 K in the upper stratosphere (due  
 483 to planetary-scale mixing). This structure mirrors the two-peaked mixing efficiency struc-  
 484 ture obtained by Gupta et al. (2021) and shown in Figure 1(b). Substantially weaker eddy  
 485 flux is found in the 550-650 K region — the region where negative  $\mu_{mix}$  is obtained us-  
 486 ing Equation 2 — but the flux is always equatorward. Weak mixing in this region is con-  
 487 sistent with past studies (e.g. Shuckburgh et al. (2001)) which employ other metrics to  
 488 quantify stratospheric mixing over a shorter time period.

489 The dashed black contours in Figure 4(a) show the corresponding eddy diffusiv-  
 490 ity,  $\mathcal{D}_{eff}$  (defined in Equation 7). Focusing on the winter hemisphere, large values of the  
 491 eddy diffusivity around the turnaround latitude (solid black) indicates that most of the  
 492 mixing is indeed localized around the turnaround latitude. This validates the use of the  
 493 turnaround latitude to compute the midlatitude mixing flux. The diffusivity is substan-  
 494 tially lower within the tropical pipe indicating minimal mixing of midlatitude air into  
 495 the deep tropics (15°S to 15°N).

496 The wintertime eddy diffusivity is overlaid on to the age-of-air profile on the 550  
 497 K isentrope using a thick orange highlighting in Figure 4(b), an isentrope height asso-  
 498 ciated with negative mixing (Figure 1). The bold orange curve highlights the latitudes



**Figure 4.** (a) Eddy flux of age,  $F_{eddy}$  (in color), and eddy diffusivity  $\mathcal{D}_{eff}$  (in dashed), for idealized model stratosphere. The solid black curve traces the region with net zero diabatic velocity. The eddy diffusivity has units of  $\text{m}^2 \text{ s}^{-1}$  and is shown at contour intervals of  $0.75 \times 10^6 \text{ m}^2 \text{ s}^{-1}$ . (b) Zonal mean age profile on the 550 K isentropic level for the idealized model (thin orange). The solid and dashed red line in the tropics and extratropics show the weighted ages  $\Gamma_u$  and  $\Gamma_d$  respectively, and the bold orange curve in the northern extratropics highlights the region with strong eddy diffusivity, i.e., 10% and higher of the maximum diffusivity at 550 K. The vertical grey bars show the turnaround latitude, demarcating the regions of upwelling and downwelling.

499 associated with strong eddy diffusivity, defined as values 10% or higher than the max-  
 500 imum diffusivity on a given level. Most of the mixing occurs between  $30^\circ\text{N}$  and  $60^\circ\text{N}$ ,  
 501 exchanging parcels with age over a range of 5.5 years to 6.8 years. This range is over-  
 502 estimated by the mixing theory which instead assumes that the tropical parcels with age  
 503  $\Gamma_u = 4 \text{ Yr}$  (solid red bar) mix with the extratropical parcels with age  $\Gamma_d = 6.8 \text{ Yr}$  (dashed  
 504 red bars). These ages correspond to the latitudes  $15^\circ\text{N}$  and  $60^\circ\text{N}$  respectively. The age

505 difference  $\Delta\Gamma$  between the parcels being mixed between the upwelling and downwelling  
 506 regions is overestimated in the assumptions leading to Equation 2, which causes the mix-  
 507 ing flux,  $\mu_{mix}$ , to be underestimated.

508 We next demonstrate that a better estimation of the age difference mixed across  
 509 the mixing barrier is indeed crucial for quantifying the mixing flux; especially in the mid-  
 510 dle and lower stratosphere. We quantify the actual age range of parcels being mixed near  
 511 the partition boundary with the  $\Gamma$ - $\theta$  circulation,  $\psi$  (defined in Equation 4) was computed  
 512 for the dry dynamical core FV3 using daily samples over a 3300-day period. The circu-  
 513 lation  $\psi$  at 450 K for FV3 is shown in Figure 5(a). Latitude and ages with a non-zero  
 514 circulation strength (in color) show the latitudes that exhibit tracer transport by mid-  
 515 latitude eddies and the age of parcels being transported at those latitudes. In the north-  
 516 ern hemisphere, a positive  $\psi$  signifies a poleward mass transport. Figure 5(a) shows the  
 517 predominantly bidirectional transport of mass in the midlatitudes in FV3. For age val-  
 518 ues less than the mean isentropic age (solid black), a positive (teal colored) circulation  
 519 cell means that the eddies transport younger age poleward. Likewise, for ages older than  
 520 the mean age, a negative (brown colored) circulation cell means that the eddies trans-  
 521 port older age equatorward.

522 The range of age values being mixed is the highest in the winter midlatitudes, con-  
 523 sistent with the predominant wave-mixing over a large spatial scale. Moreover, the cir-  
 524 culation peaks in the vicinity of the turnaround latitude. The circulation in the vortex  
 525 region is considerably weaker, on account of the vortex being isolated from the midlat-  
 526 itudes. The age range of the streamfunction is markedly narrower in the subtropical lat-  
 527 itudes, which are characterized by weak adiabatic mixing, virtually going to zero at the  
 528 equator. The distribution in the summer midlatitudes (not shown) is qualitatively sim-  
 529 ilar to the winter midlatitudes, except that mixing at higher altitudes occurs over a nar-  
 530 rower age range, i.e., over a narrower range of latitudes.

531 Focusing around the turnaround latitude in the winter hemisphere (dashed black  
 532 vertical bars;  $\approx 37.4^\circ$  N) — where the mean isentropic age is 5.2 Yr (solid black) — the  
 533 poleward transport of mass (teal) carries parcels with ages between 3 to 5.2 Yr from the  
 534 subtropics to higher latitudes. Likewise, the equatorward flux (brown) carries parcels with  
 535 ages between 5.2 Yr to 6.5 Yr from higher latitudes into the subtropics. Nearly all of the  
 536 circulation near the turnaround latitude is associated with carrying parcels with age in

537 the 4-6 Yr range, implying that a majority of parcels being carried poleward have age  
 538 between 4 and 5.2 Yr, and the majority of parcels being carried equatorward have age  
 539 between 5.2 to 6 Yr. This is seen more clearly in Figure 5(b) which shows a cross-section  
 540 of  $\psi$  along the dashed black bar in Figure 5(a) at the turnaround latitude of  $37.4^\circ\text{N}$  (blue  
 541 curve); by fixing both  $\theta$  and  $\phi$ ,  $\psi$  is now a function of age alone. The asymmetric cir-  
 542 culation distribution strongly tapers off away from the mean isentropic age of 5.2 Yr in  
 543 either direction. It confirms that most of the mixing at this latitude occurs among parcels  
 544 with age not younger than 4 Yr but also not older than 6 Yr.

545 The average age transported from the subtropics to the midlatitudes (and vice versa)  
 546 can be computed by calculating the  $\psi$ -weighted mean age exclusively over the region of  
 547 positive (negative)  $\Gamma$ - $\theta$  circulation. Using the terminology introduced in Section 2, this  
 548 weighted age-of-air actually being mixed at 450 K represented as  $\Gamma_{u,eff}$  (and  $\Gamma_{d,eff}$ ) is  
 549 shown using the solid (dashed) green vertical bars in Figure 5(b). Here,  $\Gamma_{u,eff} = 4.2$  Yr  
 550 and  $\Gamma_{d,eff} = 5.6$  Yr. Thus, the effective age difference mixed across the barrier is  $\Delta\Gamma_{eff} =$   
 551  $\Gamma_{d,eff} - \Gamma_{u,eff} = 1.4$  Yr. This age difference is approximately half of the age difference  
 552  $\Delta\Gamma = 2.5$  Yr between the ages  $\Gamma_u = 3$  yr (solid red) and  $\Gamma_d = 5.5$  yr (dashed red) as-  
 553 sumed while deriving Equation 2. This correction implies that the bulk mixing flux  $\mu_{mix}$   
 554 underestimates the actual exchange flux by about 50%. Accordingly, the perturbation  
 555 age  $\delta\Gamma_u = \Gamma_{u,eff} - \Gamma_u = 1.2$  Yr, and  $\delta\Gamma_d = \Gamma_d - \Gamma_{d,eff} = -0.1$  Yr are obtained.

556 We can use the effective mixing ages to quantify the range of mixing. Referring to  
 557 the mean age in subplot (a), the inferred values for  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$  suggest that the  
 558 mixing predominantly occurs between  $25^\circ\text{N}$  and  $55^\circ\text{N}$ , instead of  $15^\circ\text{N}$  and  $55^\circ\text{N}$  as the  
 559 red bars would indicate. Mixing is localized near the mixing barrier and the deep trop-  
 560 ics are fairly isolated from the mixing flux originating in the midlatitudes.

561 Figure 5(c) shows the  $\Gamma$ - $\theta$  circulation at the 550 K isentropic height. At this level,  
 562  $\Gamma_{u,eff} = 5.1$  Yr and  $\Gamma_{d,eff} = 6.3$  Yr, and thus  $\Delta\Gamma_{eff} = 1.2$  Yr. With  $\Gamma_u = 4.2$  Yr and  $\Gamma_d =$   
 563  $6.6$  Yr,  $\Delta\Gamma = 2.4$  Yr. Similar to 450 K, at 500 K the corrected age difference  $\Delta\Gamma_{eff}$  is  
 564 approximately half of  $\Delta\Gamma$ , and perturbation age  $\delta\Gamma_u = 0.9$  Yr and  $\delta\Gamma_d = 0.3$  Yr are ob-  
 565 tained.

566 The procedure was repeated for all the isentropic levels, i.e., the  $\Gamma$ - $\theta$  circulation was  
 567 used to compute the distribution of ages being mixed at the turnaround latitude in the  
 568 winter hemisphere. The distribution was used to estimate  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$ , which were

569 ultimately used to obtain the perturbation ages  $\delta\Gamma_u$  and  $\delta\Gamma_d$  at each vertical level. This  
 570 allows us to calculate all the variables except  $\mu_{mix}^T$  in the refined mixing equation (Equa-  
 571 tion 3), and ultimately compute  $\mu_{mix}^T$  as a residual.

572 The structure of the “effective” upwelling and downwelling age-of-air being exchanged  
 573 is shown in Figure 6(a) (black curves). The analysis using the  $\Gamma$ - $\theta$  circulation reveals that  
 574 influence of meridional gradients to the mixing estimation is the strongest in the lower  
 575 stratosphere in the tropics and in the middle stratosphere in the extratropics. The weighted  
 576 age  $\Gamma_u$  (solid orange) and the effective mixing age  $\Gamma_{u,eff}$  (solid black) in Figure 6(a) dif-  
 577 fer the most between 400 K and 600 K, and thus  $\delta\Gamma_u$  peaks in this interval. The cor-  
 578 responding midlatitude ages (dashed curves), however, differ the most between 500 K  
 579 to 900 K, and so  $\delta\Gamma_d$  peaks in this interval.

580 Our analysis also shows prominent differences between the original age difference  
 581  $\Delta\Gamma$  and the effective mixing age difference  $\Delta\Gamma_{eff}$  (Figure 6(b)). Throughout the lower  
 582 and middle stratosphere, a significant reduction in the age difference is observed, indi-  
 583 cating that the difference in the mean poleward and the mean equatorward moving age  
 584 is indeed much lower than the assumed mixing age difference  $\Delta\Gamma$ . It should be noted that  
 585 the two measures are found to be almost indistinguishable in the upper stratosphere, but  
 586 elsewhere, the true age difference being mixed is overestimated by at least 50%. Note  
 587 that this has no implications for the diabatic circulation strength, which is still directly  
 588 related to  $\Delta\Gamma$  (Linz et al., 2016).

589 Finally, we consider the new “entrainment freshening” term (C4) in the refined mix-  
 590 ing equation and assess its contribution to the net aging in the tropics. The quantity,  
 591 shown in Figure 6(d), is consistent with a peak in  $\delta\Gamma_u$ , and contributes most to the ag-  
 592 ing in the lower stratosphere. Note the negative sign in front of C4 in Equation 3, mean-  
 593 ing that a positive C4 leads to a reduction in aging. The key is that the air being ex-  
 594 pelled from the tropics is older than the mass flux weighted age  $\Gamma_u$ : entrainment “fresh-  
 595 ens” the pipe by ejection of older air. The term is positive even around 600 K, where  
 596 very weak (and even negative mixing) was estimated for the FV3 core (Figure 1). The  
 597 positive value of the term C4 in this region can explain the spurious “negative” mixing  
 598 observed in the bulk flux. With a lower stratospheric peak of 0.013 yr/K, this entrain-  
 599 ment freshening term significantly reduces the overall aging. Comparing this term with  
 600 the net vertical aging in Figure 3(c) and the advection term in Figure 3(d) reveals that

601 the entrainment freshening term C4 contributes a substantial fraction to the net aging  
 602 in the tropics. In fact, in the 400-500 K range, the magnitude of this term is more than  
 603 half as large as the net aging (comparing the green curve in Figure 6(d) and the teal curve  
 604 in Figure 3(c)), reaffirming our hypothesis.

605 We now compare the “bulk” mixing fluxes obtained from Equation 2 with the “true”  
 606 mixing fluxes obtained from Equation 3. To do this, the mixing efficiency  $\epsilon$ , which mea-  
 607 sures the strength of the mixing flux relative to the net poleward flux, is used:  $\epsilon = \mu_{mix}/\mu_{net}$ .  
 608 Using the refined mixing equation to estimate the mixing fluxes leads to a striking en-  
 609 hancement in the mixing efficiency in FV3 (Figure 7(c)). A positive age perturbation  
 610  $\delta\Gamma_u$  in the region leads to a positive offset in the mixing flux. Moreover, a markedly lower  
 611 age difference  $\Delta\Gamma_{eff} < \Delta\Gamma$  results in a multiplied (magnified) mixing flux  $\mu_{mix}^T$ . The  
 612 entrainment freshening term (C4) ensures a non-negative mixing flux throughout the strato-  
 613 sphere, while the adjustment to  $\Delta\Gamma_{eff}$  increases the amplitude of the flux. The origi-  
 614 nal mixing equation yielded an erroneously weaker  $\epsilon$  (thin dashed red) in the lower strato-  
 615 sphere than the  $\epsilon$  computed from the refined mixing equation (thick solid red). As will  
 616 be discussed in more detail later, the revised mixing efficiency (and thus the mixing flux)  
 617 bears a much stronger resemblance to the mixing efficiency obtained from the compre-  
 618 hensive model WACCM.

## 619 4.2 Estimating the meridional extent of adiabatic mixing

620 We now address the question that naturally follows: What then is the region over  
 621 which the air is truly mixed in the winter stratosphere? The estimates  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$   
 622 of the true ages of the parcels mixed across the turnaround latitude can be mapped to  
 623 the latitudes with the corresponding mean age  $\Gamma$ . That is, we find the latitudes  $\phi_1(\theta)$   
 624 and  $\phi_2(\theta)$  such that the zonal mean age  $\Gamma(\phi_1(\theta), \theta) = \Gamma_{u,eff}(\theta)$  and  $\Gamma(\phi_2(\theta), \theta) = \Gamma_{d,eff}(\theta)$ .  
 625 The two latitudes obtained by the mapping provide the equatorward and poleward ex-  
 626 tent of the midlatitude mixing. The results for FV3 from this procedure are shown in  
 627 Figure 8(a) (orange curves and shading). Throughout much of the stratosphere, the re-  
 628 gion of mixing (orange shading) roughly spans from 20°N to 60°N. The most notable fea-  
 629 ture obtained is that the mixing region is shifted poleward in the 400-500 K interval. In  
 630 addition, above 600 K, a steady but small widening of the mixing region is noted.

631 Comparing this with the region deduced from gross upwelling ages  $\Gamma_u$  and  $\Gamma_d$  (dashed  
 632 black curves and shading), we find that the most prominent differences appear in the 400-  
 633 700 K region, where the blue curve in the subtropics aligns more strongly with the sharp  
 634 subtropical age gradient while the dashed black curve does not. This is readily observed  
 635 in the 450-500 K height range. Between 400-700 K, the Linz et al. (2021) theory assumes  
 636 the mixing to dominate over a wider region in the subtropics. However, the opposite is  
 637 suggested above 700 K. Here, Linz et al. (2021) theory assumes mixing to be restricted  
 638 to poleward of 25° N, presenting a contrast to a boundary of 20° N proposed by the re-  
 639 fined theory.

640 The poleward boundary of mixing, however, is not as sensitive to the definition used.  
 641 In the lower stratosphere, the solid blue and dashed black curves from the two defini-  
 642 tions tend to be nearly identical. Some small but notable differences emerge only above  
 643 500 K where the corrected ages suggest that the mixing is restricted to relatively lower  
 644 latitudes than that expected by the original Linz et al. (2021) theory. Even though both  
 645 estimates suggest that the mixing regions are not significantly different, it is surprising  
 646 how much impact even such small differences can make to the mixing flux estimates in  
 647 the lower stratosphere; purely on account of strong sub-tropical gradients in the age-of-  
 648 air profile.

### 649 **4.3 Insights from the WACCM Comprehensive Climate Model**

650 The analysis was repeated for WACCM, which provides a more Earth-like repre-  
 651 sentation of stratospheric variability and seasonal cycle. The northern winter months of  
 652 November to March were considered for a period of five years and daily samples were  
 653 used to compute the age-of-air and the  $\Gamma$ - $\theta$  circulation. The more limited data, coupled  
 654 with more internal variability, leads to more sampling uncertainty in these calculations.

655 We first emphasize that the zonal mean age-of-air in WACCM (shown later in Fig-  
 656 ure 8(b)) is much younger than in FV3 both on account of a faster circulation and a higher  
 657 tropopause, and because the age in WACCM is computed w.r.t. the tropical stratosphere,  
 658 i.e., the whole troposphere is the source. In FV3 the age-of-air is computed w.r.t the source  
 659 region of 700 hPa and below, due to which the age at the tropical stratosphere has a mean  
 660 value of approx. 1 year. This offset does not affect the results in this study, as the dif-  
 661 ferences in age are dominated by the differences in circulation.

662 The  $\Gamma$ - $\theta$  circulation in WACCM at the 450 K isentropic height is shown in Figure  
 663 5(d). Comparing subplots (a) and (d), the gross structure of  $\psi$  is quite similar between  
 664 the idealized dry dynamical core and WACCM. The circulation peaks in the midlatitudes  
 665 and is weaker elsewhere. Moreover, for latitudes with considerable circulation strength,  
 666 parcels with age younger than the mean age (solid black) are carried poleward and parcels  
 667 with age older than the mean age are carried equatorward. Little to no circulation is noted  
 668 in the polar regions on this isentropic surface.

669 The cross section of  $\psi$  at 450 K along the turnaround latitude of 32.6°N in the win-  
 670 ter hemisphere (dashed black bar in Figure 5(d)) is shown in Figure 5(e). Yet again, we  
 671 find that the gross structure of  $\psi$  is very similar between WACCM and FV3 (Figure 5(b)).  
 672 The  $\Gamma$ - $\theta$  circulation is informative of the broad range of ages of parcels being mixed around  
 673 the barrier. At 450 K, parcels as young as 0.5 Yr are mixed with parcels with age up to  
 674 2.5 Yr. A considerable portion of the mixing, however, occurs between parcels with age  
 675  $\Gamma_{u,eff} = 0.9$  Yr and  $\Gamma_{d,eff} = 2$  Yr (solid and dashed green respectively). Computations  
 676 yield  $\delta\Gamma_u = -0.05$  Yr and  $\delta\Gamma_d = 0.75$  Yr.

677 The most conspicuous difference between the mixing in FV3 and WACCM on 450  
 678 K is that in FV3,  $\Gamma_u$  and  $\Gamma_{u,eff}$  are quite different, while  $\Gamma_d$  and  $\Gamma_{d,eff}$  are almost iden-  
 679 tical (Figure 5(b)). In contrast, for WACCM, it is the  $\Gamma_u$  and  $\Gamma_{u,eff}$  which are almost  
 680 identical and the downwelling ages  $\Gamma_d$  and  $\Gamma_{d,eff}$  which are quite different. So, the (rel-  
 681 atively weaker) subtropical gradients barely impact the estimation of the tropical age  
 682 being mixed. Nevertheless, the  $\Gamma$ - $\theta$  circulation in both the models suggest that the true  
 683 age difference mixed around the turnaround latitude is much lower than the bulk age  
 684 difference  $\Delta\Gamma$ , with most of the difference coming from the downwelling ages.

685 Next, the  $\Gamma$ - $\theta$  circulation at 550K is shown in Figure 5(f). At this level, parcels with  
 686 stratospheric age as young as 2 Yr are mixed with parcels with age as old as 4 Yr. How-  
 687 ever, the bulk of the mixing occurs between ages  $\Gamma_{u,eff} = 2.4$  Yr and  $\Gamma_{d,eff} = 3.4$  Yr,  
 688 yielding an effective mixed age difference  $\Delta\Gamma_{eff} = 1$  Yr, much less than a  $\Delta\Gamma = 1.4$  Yr.  
 689 It follows that  $\delta\Gamma_u \approx 0$  and  $\delta\Gamma_d = 0.4$  Yr. Similar to 450 K, a weak perturbation age  
 690  $\delta\Gamma_u$  on 550 K is obtained for WACCM.

691 Similar to FV3, the wintertime averages of all the terms in the refined mixing equa-  
 692 tion were computed for WACCM. Here, we only show the effective age difference  $\Delta\Gamma_{eff}$   
 693 (Figure 6(c)) and the entrainment freshening term C4 for WACCM (violet curve in Fig-

694 ure 6(d)); terms which are associated with the revised mixing equation. Our estimate  
 695 for  $\Delta\Gamma_{eff}$  using the  $\Gamma$ - $\theta$  circulation shows a considerably smaller age difference being mixed  
 696 across the turnaround latitude; Therefore, for both FV3 and WACCM,  $\Delta\Gamma_{eff} \ll \Delta\Gamma$ .

697 The entrainment freshening term for WACCM (violet curve in 6(d)) is weaker and  
 698 unlike FV3, barely contributes to the net vertical aging in the lower stratosphere. This  
 699 is due to the fact  $\Gamma_u \approx \Gamma_{u,eff}$  in WACCM; the air being entrained into the extratrop-  
 700 ics carries the mean age with it. In fact, the quantity is nearly vanishing throughout the  
 701 stratosphere, indicating that entrainment freshening does not have a strong effect on the  
 702 mixing flux estimation in WACCM. Since C4 is weakly negative in the lower stratosphere,  
 703 a smaller mixing term (C3) leads to a slight reduction in the “true” mixing flux estimated  
 704 from Equation 3. We believe, however, that the entrainment mixing term is not suffi-  
 705 ciently sampled due to limited data availability for WACCM. The  $\Gamma - \theta$  circulation is  
 706 sensitive to the sampling frequency and sample size, more so than the other dynamical  
 707 quantities in the revised mixing equation. Sampling the streamfunction at a frequency  
 708 higher than the timescales associated with isentropic mixing is crucial to obtaining a smooth  
 709 streamfunction profile. Since the entrainment freshening term is weak and not always  
 710 physically consistent, we neglect this term and only consider the effective age difference  
 711 while computing the revised mixing efficiency for WACCM.

712 The figure shows the bulk and true mixing efficiency computed for WACCM. Both  
 713 the bulk and true mixing efficiency in WACCM (in dashed and bold gray respectively)  
 714 have similar gross structure throughout the stratosphere. The two mixing profiles dif-  
 715 fer in that the true mixing is more enhanced in the lower stratosphere.  $\Delta\Gamma_{eff}$  which is  
 716 smaller than  $\Delta\Gamma$  results in enhancement of the mixing fluxes in lower and middle strato-  
 717 sphere. A stronger mixing in WACCM is also consistent with a stronger wintertime cir-  
 718 culation in WACCM. The bold grey curve suggests a mixing efficiency of 2 in the lower  
 719 stratosphere. This is equivalent to the true mixing flux  $\mu_{mix}$  being two-thirds the strength  
 720 of the equatorward mass flux.

721 The same procedure as described in Section 4.2 was applied to WACCM in order  
 722 to estimate the latitudinal span of wave-induced mixing. Figure 8(b) shows the mixing  
 723 span estimated for WACCM using both  $\Gamma_{u,eff}, \Gamma_{d,eff}$  (orange curves and shading) and  
 724  $\Gamma_u, \Gamma_d$  (dashed black curves and shading). The two types of ages yield regions which are  
 725 different in structure. The region determined from corrected ages spans from 20°N to

726 45°N in the lowermost stratosphere (400K) and gradually decreases in width up to 700  
 727 K where it only spans from 15°N to 35°N. Most notable is the equatorward shift in the  
 728 orange curve in the subtropics, in phase with the turnaround latitude (solid black). Above  
 729 700 K, the region re-widens, spanning all the way from the equator on the left to 60°N  
 730 on the right. We note that above 1100 K the corrected tropical age  $\Gamma_{u,eff}$  is younger than  
 731 the equatorial age and thus the mixing span has been truncated at the equator. Such  
 732 young ages are most likely obtained due to (i) rapid equatorward shift of the turnaround  
 733 latitude with height, (ii) transience in the zonal mean age at lower latitudes in WACCM,  
 734 and (iii) prevailing inter-hemispheric transport.

735 In contrast, the mixing span determined from  $\Gamma_u, \Gamma_d$  spans from 20°N on the equa-  
 736 torward side to 70°N on the poleward side. This range is considerably wider than that  
 737 for FV3 (black shading in subplot (a)). Interestingly, for FV3, both the dashed black curves  
 738 lay on either side of the turnaround latitude. This is not the case for WACCM, where  
 739 the turnaround latitude makes a drastic equatorward turn. Since the corrected ages were  
 740 inferred by analyzing the  $\Gamma$ - $\theta$  circulation around the turnaround latitude, differences in  
 741 the diabatic velocity structure between FV3 and WACCM lead to major differences ob-  
 742 tained between the orange and black shaded regions as well in WACCM. Despite these  
 743 differences, it is found that the deep tropics barely witness any wave-induced adiabatic  
 744 mixing and are fairly isolated, just as in FV3.

745 In summary, the structure of the  $\Gamma$ - $\theta$  circulation is similar for FV3 and WACCM,  
 746 and both models demonstrate an overestimation of the mixed age difference in the orig-  
 747 inal mixing equation. Employing the refined mixing theory, which estimates the age dif-  
 748 ference more accurately, yields similar mixing efficiency  $\epsilon$  for the two models. Limited  
 749 sampling in WACCM may also contribute slightly to the observed differences between  
 750 FV3 and WACCM. Only 5 years of daily data, 150 days per year, was considered from  
 751 WACCM. This is significantly lower than 10 years of data, 360 days per year, consid-  
 752 ered for the dynamical core. Moreover, the original mixing equation of Linz et al. (2021)  
 753 assumes a stratosphere in steady state, and for WACCM the NDJFM climatology is cou-  
 754 pled with the effects of dynamics and transport during other seasons. The choice of per-  
 755 petual solstice climatology in FV3 ensures that contributions from inter-seasonal vari-  
 756 ability are eliminated in the analysis and the circulation is statistically steady. Some dif-  
 757 ferences in mixing may also be due to the presence of a nudged QBO in WACCM vs per-  
 758 sistent tropical easterlies in FV3.

## 5 Discussion and Conclusions

Wave-induced adiabatic mixing plays a key role in mixing trace gases over large spatial scales in the winter stratosphere, and in influencing their global distribution. Accurately quantifying this mixing is key to understanding transport trends in the stratosphere in a changing climate. We have proposed a method to improve the estimates of the adiabatic mixing flux in the stratosphere obtained from applying the vertical age gradient theory of Linz et al. (2021), and have used it to quantify the extent of mixing between the tropics and midlatitudes. We show that the deep tropics do not participate in the adiabatic mixing of tracers in most of the stratosphere.

The theoretical formulation of Linz et al. (2021) partitions the stratosphere into regions of diabatic upwelling and diabatic downwelling. This allowed them to connect the vertical gradient of tracer (age-of-air) in the upwelling region (tropics) with the quasi-horizontal adiabatic mixing flux between the two partitions, thus allowing computation of the mixing flux using tracer-based measures. The theory assumes fast mixing within both the tropics and the extratropics, and hence neglects any gradients in age-of-air profile expected near the subtropical transport barrier. We show that this assumption leads to an overestimation of the meridional span of adiabatic mixing in the winter stratosphere, and as a result, an underestimation of the mixing fluxes. We refined the framework in order to include the effects of meridional age gradients. This allows a more comprehensive estimation of the age of the air parcels exchanged between the upwelling and downwelling partitions of the stratosphere. We used the refined age mixing theory to quantify the meridional span of adiabatic mixing in the winter stratosphere.

The correction was made possible by a novel strategy to obtain the complete age range of air parcels mixed around the turnaround latitude through the  $\Gamma$ - $\theta$  circulation streamfunction. The streamfunction is essentially a joint distribution which allows quantification of mass transport in the latitude-age-isentrope space; i.e., at a given latitude and isentropic height, what fraction of the total mass transport is associated with a given age? Having information of the age-of-air distribution being mixed enabled estimation of the “true” ages that are mixed around the winter hemisphere turnaround latitude. We calculated the true difference in the age-of-air being transported from upwelling to the downwelling region and vice versa, and found that this effective age difference is significantly smaller than the age difference assumed in the Linz et al. (2021) theory.

791 The true ages of air being mixed was used to re-derive the mixing equation of Linz  
 792 et al. (2021) (Equation 2) to yield Equation 3, which accounts for age gradients as well.  
 793 The new mixing equation replaces the gross age differences  $\Delta\Gamma$  of the mixed parcels with  
 794 the revised age difference calculated from the  $\Gamma$ - $\theta$  circulation. It also contains an addi-  
 795 tional advection term (the entrainment freshening) that captures the effects of entrain-  
 796 ment on the tropical age in the presence of subtropical gradients.

797 For a thorough test of the proposed theory, we considered both an idealized model,  
 798 a dry dynamical core and a comprehensive model. Both of these choices were made to  
 799 maximize the error that will be introduced into the original mixing theory, as averag-  
 800 ing over several years of seasonally-varying circulation (as in Linz et al. (2021)) will damp  
 801 the effects highlighted here. Both the models exhibit similar  $\Gamma$ - $\theta$  circulation, qualitatively,  
 802 in the lower stratosphere. Further, results from both models indeed verify an overesti-  
 803 mation of the age difference  $\Delta\Gamma$  mixed between the upwelling and downwelling regions  
 804 by the original mixing equation. On applying the revised mixing theory to the models,  
 805 it was found that both the idealized and the comprehensive model qualitatively produced  
 806 very similar mixing efficiencies (a proxy for mixing fluxes).

807 The models show a strong agreement throughout the stratosphere. In fact, we high-  
 808 light that both models also agree that the deep tropics, from the equator all the way to  
 809  $15^\circ\text{N}$ , is fairly isolated from adiabatic mixing and that the equatorial region almost ex-  
 810 clusively experiences pure diabatic advection of air. This is especially true in the lower  
 811 and the middle stratosphere. Our findings corroborate the findings from an independent  
 812 study by the co-authors Curbelo and Linz, who used Lagrangian trajectories to quan-  
 813 tify the mixing of midlatitude air into the tropics.

814 We conclude that our analysis highlights both the sensitivity of the mixing flux to  
 815 the strong age gradients near the turnaround latitude, and their importance in the es-  
 816 timation of the quasi-horizontal mixing flux. It also demonstrates that tracer-based mea-  
 817 sures can be used to estimate the meridional span of adiabatic mixing in the winter mid-  
 818 latitudes given sufficiently detailed data. The theory proposed in this study can be used  
 819 to quantify mixing in the observed stratosphere as well, using both satellite observations  
 820 and reanalyses, especially in the lower and middle stratosphere. Past studies have used  
 821 transport models and specified dynamics to compute and compare the mean age-of-air  
 822 across different reanalyses (Chabrilat et al., 2018; Ploeger et al., 2019). Our analysis could

823 be further used to connect the impact of differences in adiabatic mixing to the observed  
 824 age differences, and over a longer timescale than those considered in this study, using  
 825 more than forty years of reanalyses.

826 Projecting the mass transport into latitude-age-isentrope space using the  $\Gamma$ - $\theta$  cir-  
 827 culation presents a new way to assess the adiabatic mixing structure in the stratosphere.  
 828 The streamfunction couples the dynamics and transport into a single quantity, and in-  
 829 stead of age-of-air tracer, it could be generalized to any chemical trace gas species with  
 830 weak sources and sinks in the stratosphere: nitrous oxide, sulphur hexafluoride and methane,  
 831 for example. The circulation distribution can be of value in investigating transport ex-  
 832 changes across the wintertime polar vortex, or mixing in the upper troposphere lower  
 833 stratosphere (UTLS) region where the combined effects of stratosphere-troposphere mass  
 834 exchange, convection and jet instability, influence the ozone and water vapor reservoirs.

## 835 **Appendix A Revised Mixing Equation**

836 The age-of-air is an idealized tracer with no spatial sources and sinks away from  
 837 the boundary. The age tracer only has a source in time, i.e., the age of a parcel in the  
 838 free atmosphere ages at a rate of 1 second per second. Therefore, for a stratosphere in  
 839 steady state, the tracer continuity equation for the age-of-air, following Linz et al. (2016),  
 840 can be generally written as :

$$\frac{1}{\rho_{\theta}} \nabla \cdot \mathcal{F} = 1 \quad (\text{A1})$$

841 where  $\rho_{\theta} = \frac{-1}{g} |dp/d\theta|$  is the three-dimensional isentropic density, and  $\mathcal{F}$  is the net advective-  
 842 diffusive mass flux of age. Considering a stratospheric air mass enclosed between two isen-  
 843 tropes  $\theta$  and over  $\theta + \Delta\theta$  in the vertical and spanning the whole upwelling partition in  
 844 the horizontal, the net age budget for the box (or layer) can be, most simply, written as  
 845 :

$$\text{Age Flux Out} = \text{Age Flux In} + \text{Source} \quad (\text{A2})$$

846 Mathematically, this is equivalent to multiplying Equation A1 by  $\rho_{\theta}$ , and horizon-  
 847 tally integrating both sides over the upwelling region and over the isentropes  $\theta$  and  $\theta +$   
 848  $\Delta\theta$  in the vertical. Assuming the contribution from diffusion to be small, all the fluxes

849 take an advective form. Adopting the terminology introduced in Linz et al. (2021) and  
 850 in Section 2, the total age flux entering the box is the sum of the upward directed di-  
 851 abatic flux of age entering the box from below (i.e. the  $\theta$  isentrope) and the adiabatic  
 852 advective flux of older age  $\Gamma_d - \delta\Gamma_d$  by the mixing flux  $\mu_{mix}$  from the downwelling re-  
 853 gion. Similarly, the total age flux exiting the box is the sum of the upward directed di-  
 854 abatic flux of age exiting the box from the top (i.e. the  $\theta + \Delta\theta$  isentrope) and the pole-  
 855 ward advective flux of  $\Gamma_u + \delta\Gamma_u$  by the entrainment  $\mu_{ent}$  into the downwelling region.  
 856 Both  $\mu_{mix}$  and  $\mu_{ent}$  have units of kg/s/K (mass flux per unit isentropic height). The tracer  
 857 continuity equation for the infinitesimal layer can thus be written as :

$$\Gamma_u \mathcal{M}_u(\theta + \Delta\theta) + \int_{\theta}^{\theta + \Delta\theta} \mu_{ent}(\Gamma_u + \delta\Gamma_u) d\theta = \int_{\theta}^{\theta + \Delta\theta} \mu_{mix}(\Gamma_d - \delta\Gamma_d) d\theta + \Gamma_u \mathcal{M}_u(\theta) + \int_{\theta}^{\theta + \Delta\theta} \rho_{\theta} d\theta \quad (\text{A3})$$

858 We rearrange and express  $\Gamma_u \mathcal{M}_u(\theta + \Delta\theta) - \Gamma_u \mathcal{M}_u(\theta)$  as  $\Delta\theta \cdot d(\Gamma_u \mathcal{M}_u)/d\theta$  and  
 859 apply the product rule to get the left hand side of Equation A4. In addition, we approx-  
 860 imate the integrals by multiplying the integrand by  $\Delta\theta$ . Cancelling  $\Delta\theta$  from each side,  
 861 we get :

$$\Gamma_u \frac{d\mathcal{M}_u}{d\theta} + \mathcal{M}_u \frac{d\Gamma_u}{d\theta} = -\mu_{ent}(\Gamma_u + \delta\Gamma_u) + \mu_{mix}(\Gamma_d - \delta\Gamma_d) + \sigma_u \quad (\text{A4})$$

862 where  $\sigma_u$  is the isentropic density horizontally integrated over the upwelling region (units  
 863 kg/K). By mass continuity,  $\mu_{net} = -\frac{d\mathcal{M}_u}{d\theta}$ , and by definition,  $\mu_{net} = \mu_{ent} - \mu_{mix}$ . On  
 864 substitution and rearranging, we get :

$$\frac{d\Gamma_u}{d\theta} = \frac{\sigma_u}{\mathcal{M}_u} + \frac{\mu_{mix}(\Delta\Gamma_d - \delta\Gamma_d - \delta\Gamma_u)}{\mathcal{M}_u} - \frac{\mu_{net}\delta\Gamma_u}{\mathcal{M}_u} \quad (\text{A5})$$

865 For a steady state circulation, the upwelling mixing flux balances the downwelling  
 866 mixing flux and thus,  $\mathcal{M}_u = -\mathcal{M}_d = \mathcal{M}$ . On substitution, this yields the revised mix-  
 867 ing equation A6, identical to Equation 3 :

$$\frac{d\Gamma_u}{d\theta} = \frac{\sigma_u}{\mathcal{M}} + \mu_{mix} \frac{\Delta\Gamma_{eff}}{\mathcal{M}} - \mu_{net} \frac{\delta\Gamma_u}{\mathcal{M}} \quad (\text{A6})$$

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 881 supercomputing storage. Zonal mean model data, in a polished form, is being prepared  
 882 and will be made publicly available for peer review latest by 15 September 2022.

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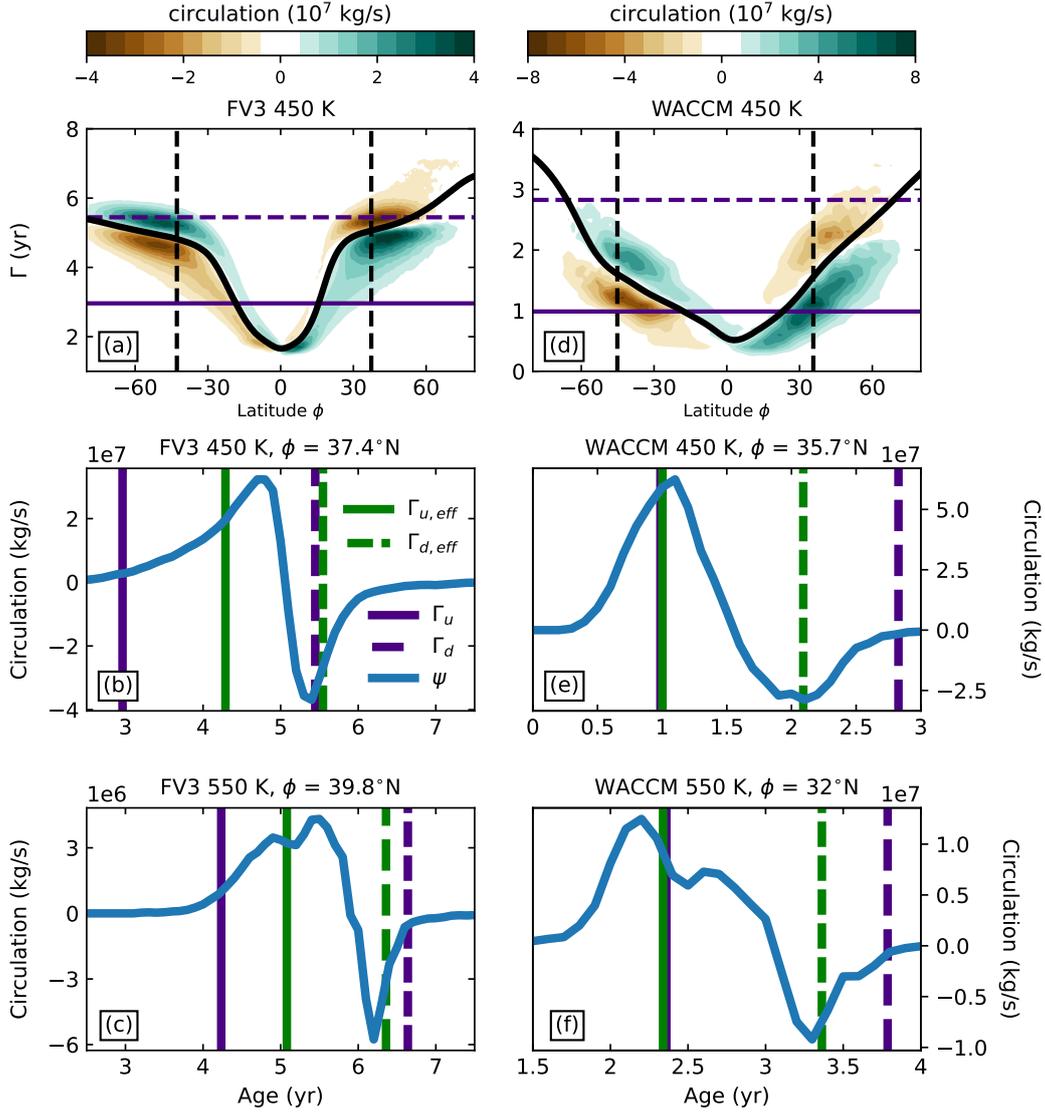
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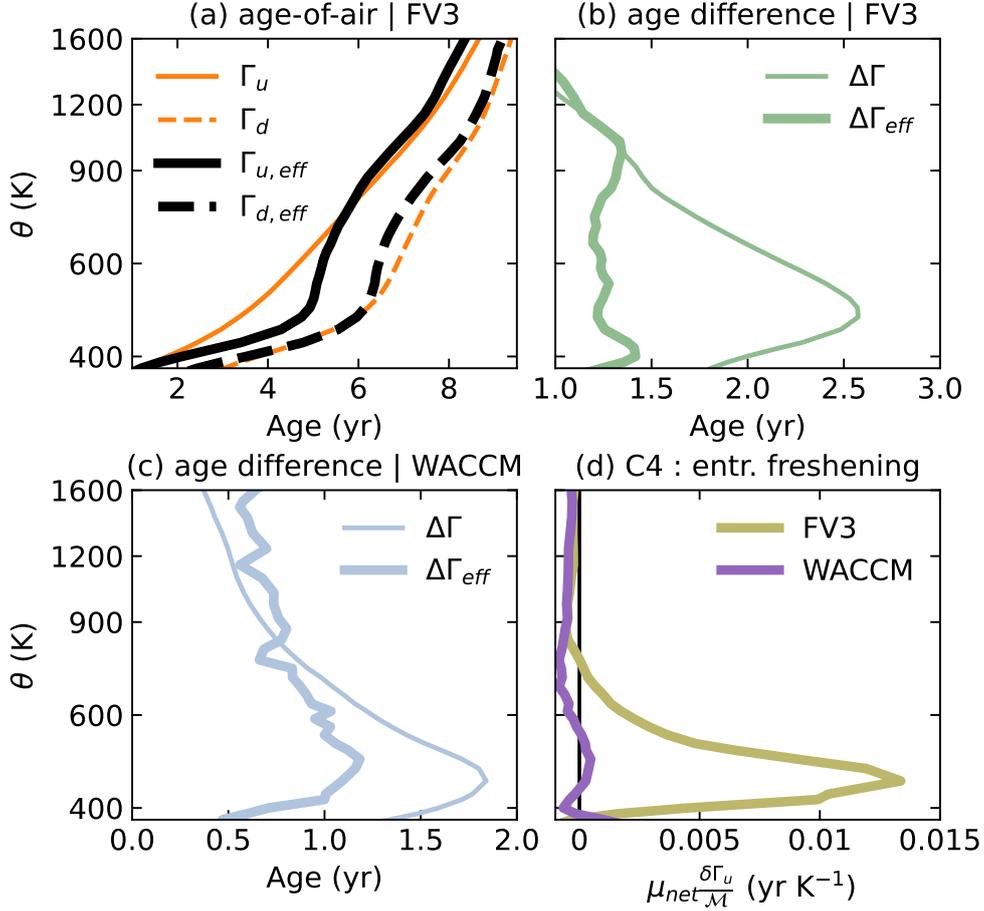
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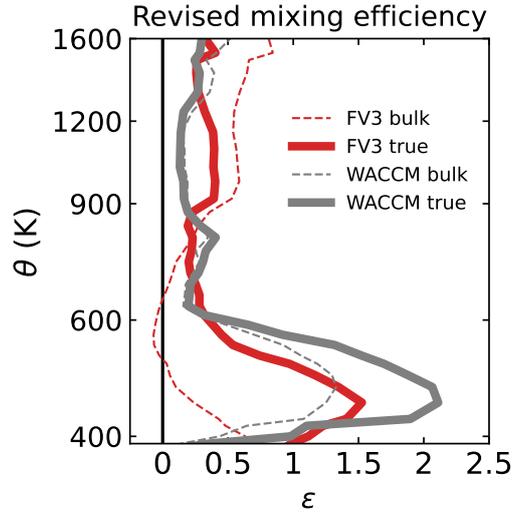
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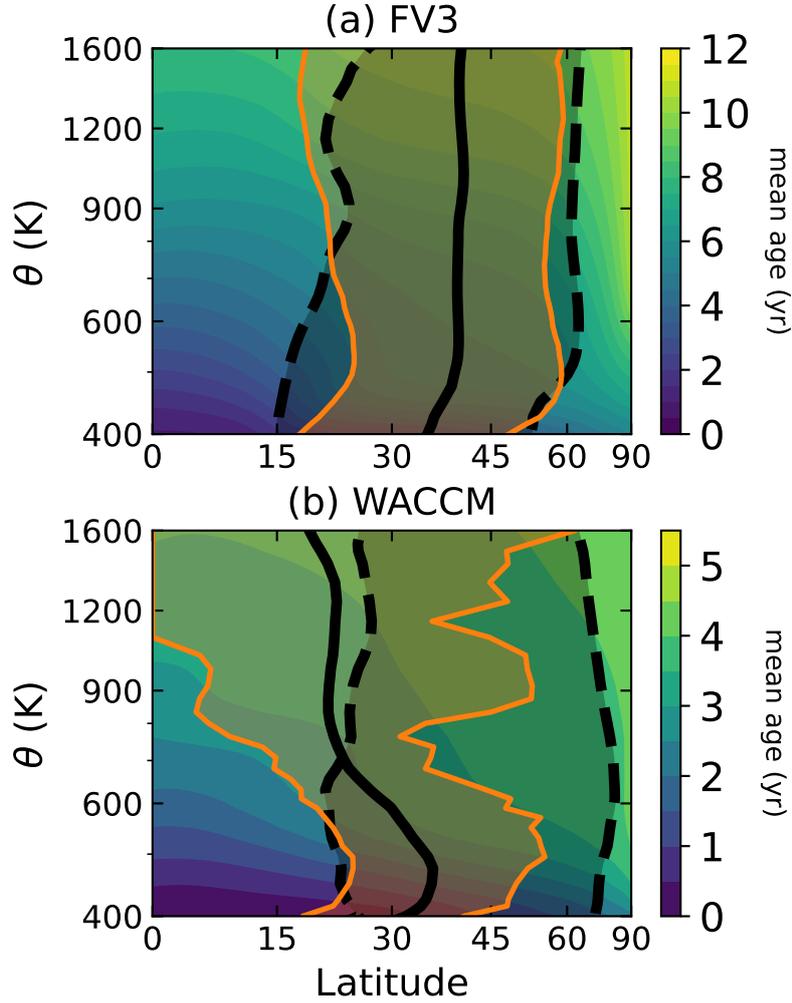
**Figure 5.** The  $\Gamma$ - $\theta$  circulation,  $\psi$ , as defined in Equation 4 for (a) the FV3 core and (d) the WACCM model, at 450 K isentropic height. In both (a) and (b), the black curves shows the mean age-of-air at 450 K, and the solid and dashed red lines respectively mark the upwelling and downwelling ages,  $\Gamma_u$  and  $\Gamma_d$  at this height. The dashed black curves in each hemisphere mark the turnaround latitudes at 450 K. Subplots (b) and (e) show a slice of  $\psi$  at 450 K in bold blue across a fixed latitude (chosen near the turnaround latitude for FV3 and WACCM respectively, i.e., they show a cross-section of  $\psi$  along the dashed black curve in (a) and (d)). Subplots (c) and (f) are identical to subplots (b) and (e) except that they show the distribution on 550 K. In subplots (b-c,e-f), the solid and dashed red vertical bars show  $\Gamma_u$  and  $\Gamma_d$  respectively, while the solid and dashed green vertical bars show the ages  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$  which are calculated from the  $\Gamma$ - $\theta$  circulation by a circulation-weighted average over the positive and negative portion of  $\psi$  (in blue). The ages  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$  more accurately reflect the ages of the parcels actually being mixed at the given height and indicate an overestimation of the age difference  $\Delta\Gamma$  being mixed across the transport barrier, as considered in Equation 2.



**Figure 6.** A comparison of the terms between the original mixing equation, Equation 2, and the revised mixing equation, Equation 3. (a) The mass-flux weighted ages  $\Gamma_u$  and  $\Gamma_d$  (in orange) and the  $\psi$ -weighted ages  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$  (in black) for the idealized FV3 model. The upwelling and downwelling ages are shown using solid and dashed curves respectively. (b) The age differences  $\Delta\Gamma$  (thin green) and  $\Delta\Gamma_{eff}$  (bold green) as inferred from the ages in subplot (a) for FV3. (c) Corresponding age differences  $\Delta\Gamma$  (thin blue) and  $\Delta\Gamma_{eff}$  (bold blue) obtained for NDJFM months in WACCM. (d) The entrainment freshening term, i.e., term C4 in Equation 3 for both the idealized FV3 model (yellow) and the comprehensive model WACCM (violet).



**Figure 7.** The bulk mixing efficiency and true mixing efficiency  $\epsilon$ , which are defined as the ratio of the bulk/true mixing flux to the net poleward flux, i.e., bulk  $\epsilon = \mu_{mix}/\mu_{net}$  and true  $\epsilon = \mu_{mix}^T/\mu_{net}$ . The mixing efficiency obtained by applying Equation 3 is shown using thick solid curves. The mixing efficiency obtained for both FV3 (red) and WACCM (grey) using the original mixing equation of Linz et al. (2021) is shown using thin dashed curves for reference.



**Figure 8.** Age inferred meridional range of adiabatic mixing in (a) the idealized FV3 core and (b) the comprehensive model WACCM. For both the plots, the zonal mean age profile is shown in color. The solid black curve shows the turnaround latitude, and the dashed black curves on either side show the latitudes with  $\Gamma_u$  and  $\Gamma_d$  as the mean age. The orange curves, and the area enclosed, demarcate the extent of the mixing region inferred from the mean ages  $\Gamma_{u,eff}$  and  $\Gamma_{d,eff}$ . These ages were estimated as an average of the circulation distribution around the turnaround latitude. The analysis reveals little-to-no mixing of the midlatitude air into the deep tropics. The deep tropics are markedly more isolated for the FV3 core, than for the WACCM model. Both the subplots use separate colorbars.