MATH-GA 2470: Ordinary Differential Equations

Lectures: Tuesday and Thursday, 12:30-1:45 pm (WWH 1302)

Edwin Gerber (epg2@nyu.edu)
Office Hours: Wednesday 1:30-3:30 pm (WWH 911)

Textbooks

Strogatz, Steven, *Nonlinear Dynamics and Chaos (Second Edition)*, Westview Press, 2015.

Guckenheimer, John and Philip Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, 1983. [**free!** https://link.springer.com/book/10.1007/978-1-4612-1140-2]

Course Description

Ordinary Differential Equations, ODE for short, was probably my least favorite class in mathematics when I was a student. It may have something to do with the fact that the class met at 8 am, or that we used a book written by the professor, but at the time I felt that there were so few equations we could solve (at least out of the universe of equations one could pose), and the solutions often depended on a few tricks, rather than something fundamental. Ugh, I really missed the point!

ODE is where we realize the power of calculus, *the ability to make predictions*. Yes, with calculus you can predict the future! Well, given the physical laws of the universe (or the markets, or your favorite system) and appropriate initial conditions, *differential equations* allow us to forecast how it will evolve. They are indispensable in my field of research, where we work with ODE's complicated sibling (*partial differential equations*, which involve multiple dimensions) to predict the response of the climate systems to external forcing, i.e., our greenhouse gas emissions.

What is a graduate ODE course? I'm admittedly trying to answer that myself. It is my understanding that undergraduate ODE should thoroughly expose you to 1-dimensional, first order ODEs (the tricks, existence and uniqueness, etc.), and familiarize you with higher order ODEs and systems of first order ODEs. The treatment of higher dimensional/order problems, however is largely linear at the undergraduate level. I expect you've seen series solutions to 2nd order ODEs with non-constant coefficients and know how to handle linear systems with constant coefficients, that you are comfortable with a phase space description of 2-d linear systems, and know how to assess the stability of linear systems and the fixed points of nonlinear systems.

Building on this background, the goal of the course is to explore nonlinear systems. We'll take a more applied track, first reviewing one dimensional flows (bifurcations and flows on the circle) and then move on to higher dimensional systems, 2 dimensions, where we can explore oscillations and limit cycles, and then 3 dimensions and beyond, where we can explore chaos and fractals.

I will try to motivate the mathematics with a few well known systems, including Van Der Pol's equation, Duffing's equation, and the Lorenz 1963 "butterfly" system. We

will also explore numerical solutions to ODEs, which allow us to gain more insight about complex systems.

Grades

Biweekly Homework	50%
Final Exam or Project	50%

Policies

In keeping with University standards, I will not require attendance — **please stay home if you are feeling ill or otherwise believe you may pose risk to others! Mask are always welcome.** To help students unable to come in-person, I will will share my lecture notes. I appreciate it if you let me know when you cannot attend class, and please contact me if you have to miss more than a couple classes due to illness, so we can ensure you don't fall behind..

And finally, a word on academic integrity. **Do not cheat.** I will require an academic pledge on the exams. If you work with others on homework (which I encourage), please just note your partners. **If you are caught cheating, the penalty will be a zero for the assignment/exam, failing the course, and/or reporting you to the Office of the Dean, depending on the severity of the infraction.**

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. The Moses Center website is www.nyu.edu/csd. Please contact the Moses Center for Students with Disabilities (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.

On respect and inclusion

As an instructor, I will strive to create a safe, respectful, and inclusive environment for all students regardless of their identity. I recognize and value diversity inside and outside of the classroom, and recognize that each student has a unique contribution to make and brings with them different strengths and weaknesses. I welcome your ideas for how to promote a better understanding and deeper learning in this class as a community. Please feel free to ask questions, to participate in discussions, and to suggest new approaches to the class content. Please also feel welcome to raise any issue you may have in class or outside of class, including reporting incidents of bias or discrimination, whether intentional or unintentional, either to me, to your advisor(s)/mentor(s), or by using the NYU Bias Response Line.