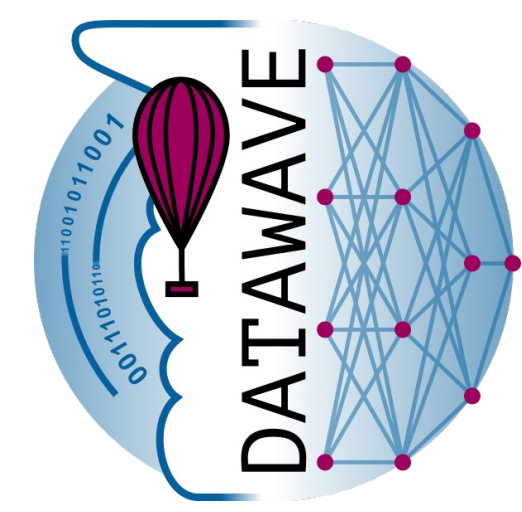


# A 1-D QBO model testbed for data-driven gravity wave parameterization: Generalization and calibration

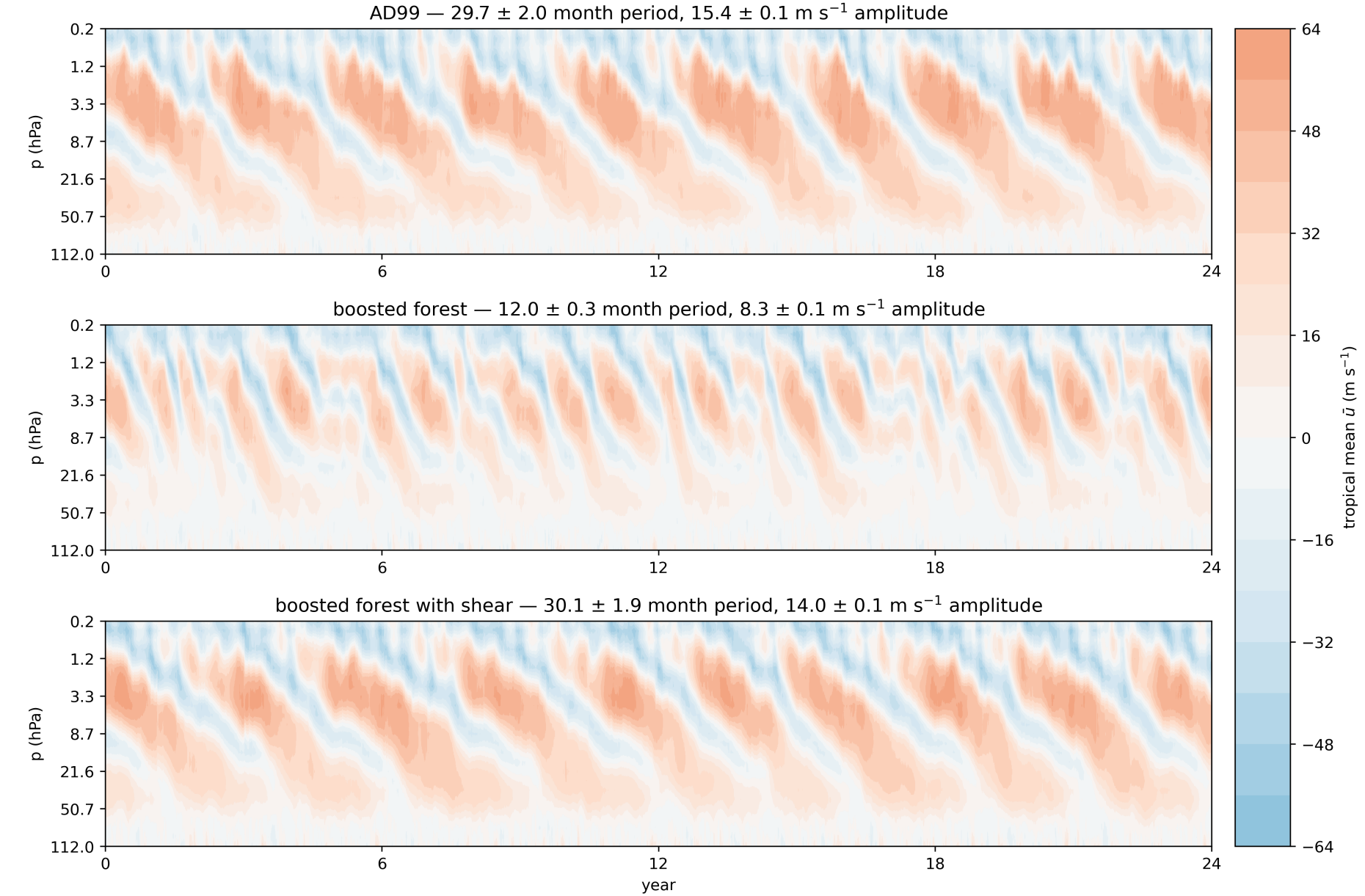
Ofer Shamir<sup>1</sup>, Zihan Shao<sup>1</sup>, L. Minah Yang<sup>1</sup>, David S. Connelly<sup>1</sup>, Steven Hardiman<sup>2</sup>, Edwin P. Gerber<sup>1</sup>

<sup>1</sup>Courant Institute of Mathematical Sciences, New York University, New York, New York. <sup>2</sup>Met Office Hadley Centre, Met Office, FitzRoy Road, Exeter EX1 3PB, UK



## (1) Motivation

- A key metric of gravity wave (GW) parameterization tuning is the fidelity of the simulated Quasi-Biennial Oscillation (QBO).
- Simulated QBOs in an intermediate complexity atmospheric model (MiMA), forced with emulators of physics-based GW parameterization (AD99<sup>1</sup>), are highly variable.



- Sensitivity analysis of the QBO response to external forces (e.g., CO<sub>2</sub>) and GW parameters is computationally taxing.
- We explore the generalization and calibration of data-driven GW parameterization in a 1D QBO model testbed.

## (2) Model and stochastic wave forcing

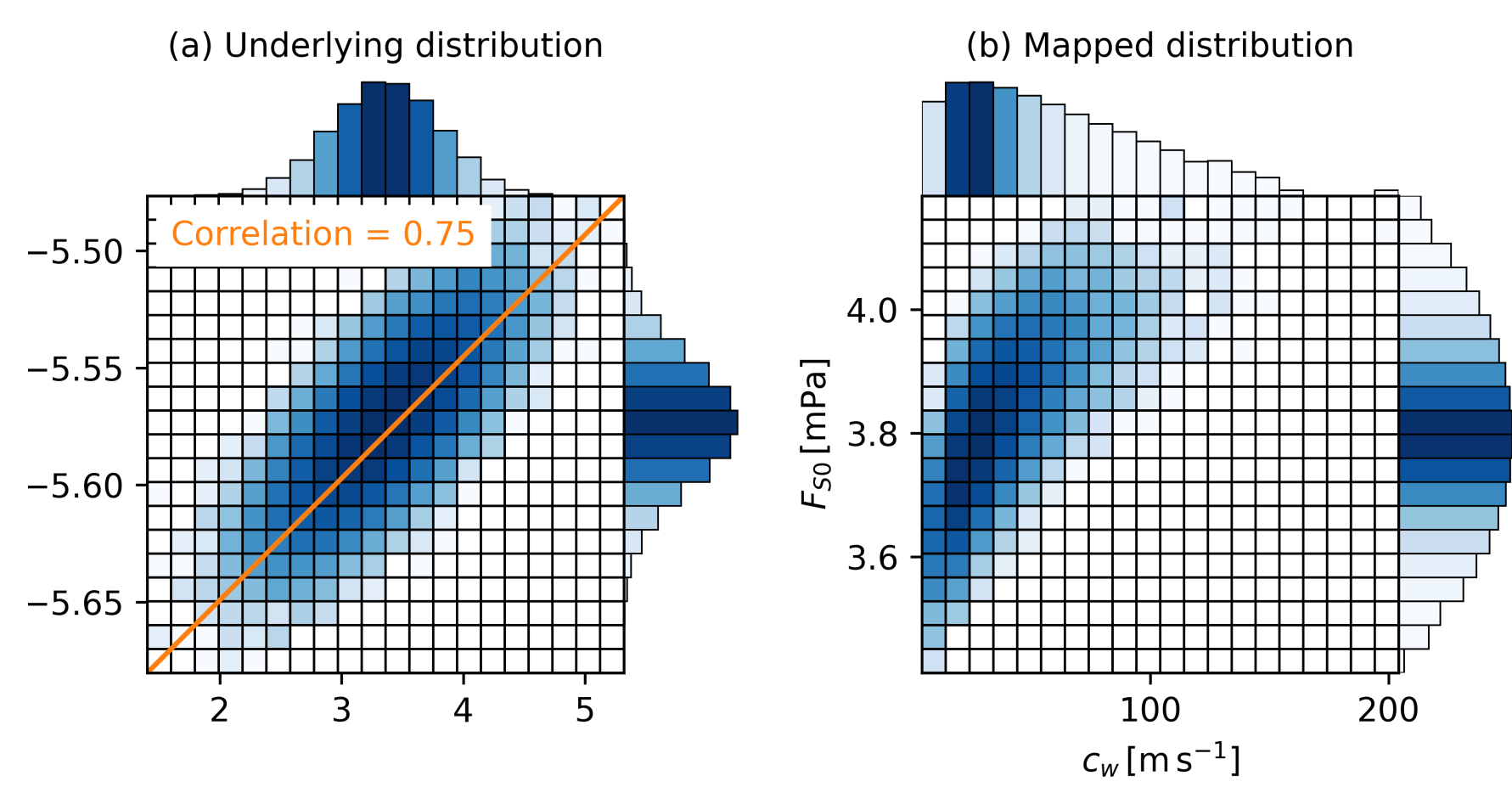
- A hybrid of the 1D QBO models studied in HL72<sup>2</sup> and P77<sup>3</sup>, forced by a collection of monochromatic waves packets:

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - \kappa \frac{\partial^2 u}{\partial z^2} = -\frac{1}{\rho} \frac{\partial}{\partial z} \sum_i A_i \exp \left\{ -\int_{z=z_1}^z \frac{\alpha(z') N}{k_i (u - c_i)^2} dz' \right\}$$

- The wave spectrum follows AD99<sup>1</sup>:

$$A(c) \propto \text{sgn}(c) \exp \left[ -\ln 2 \left( \frac{c}{c_w} \right)^2 \right]$$

- We add stochasticity to the wave forcing:** at each time step the total source flux  $F_{S0} = \sum_i |A_i|$  and spectral width  $c_w$  are drawn from a bi-variate log-normal distribution.

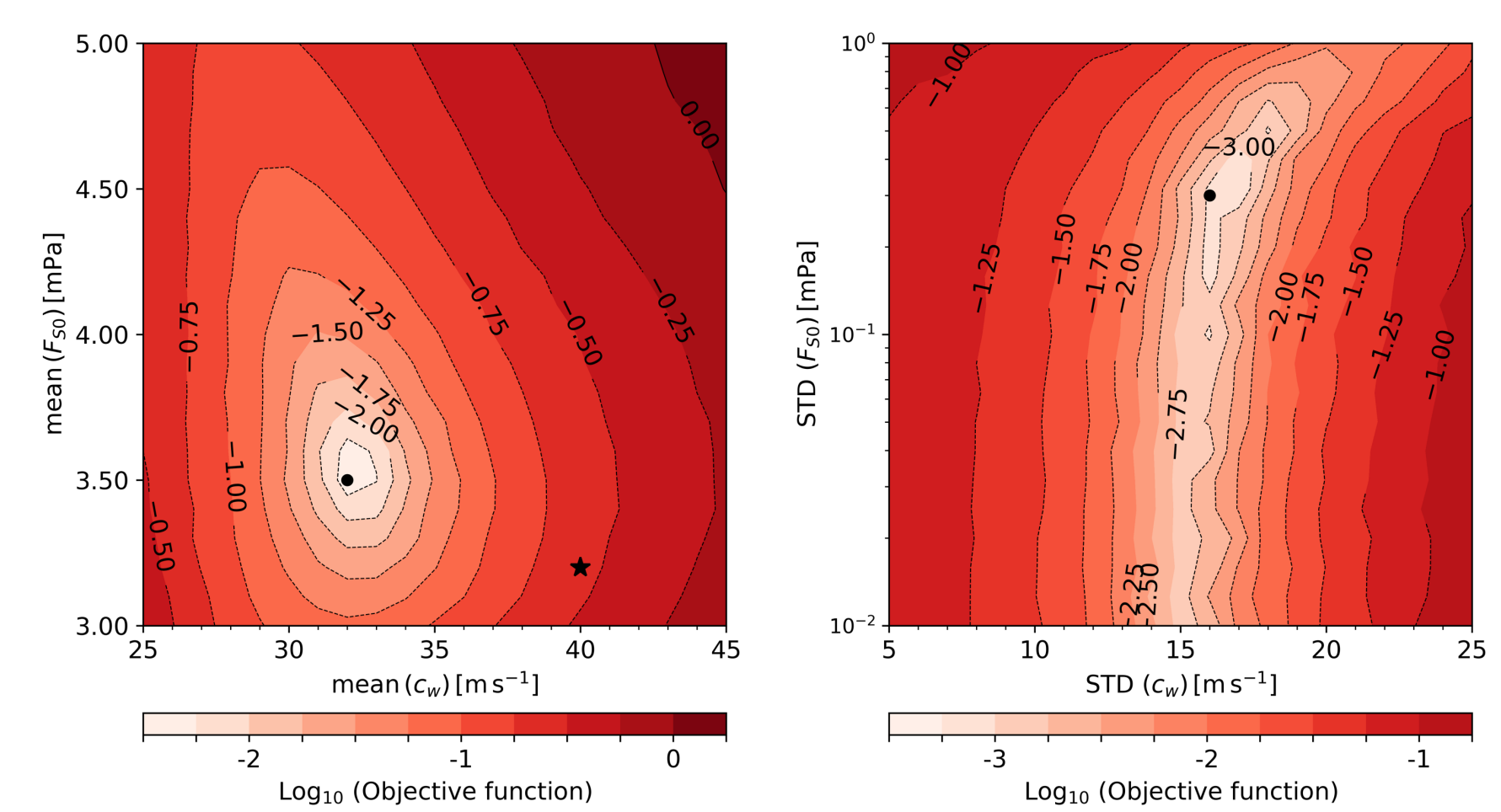


- Physically,  $F_{S0}$  (related to precipitation<sup>2</sup>) and  $c_w$  (related to convection depth) are positively correlated.

## (3) “Optimal” / “observed” wave forcing

- The control GW spectrum corresponds to the unique combination of wave flux and spectral width that yields the “observed” QBO amplitude ( $\sigma$ ) and period ( $\tau$ ) according to (\*).

$$\frac{[\sigma(25 \text{ km}) - 33 \text{ m/s}]^2}{[33 \text{ m/s}]^2} + \frac{[\sigma(20 \text{ km}) - 19 \text{ m/s}]^2}{[19 \text{ m/s}]^2} + \frac{[\tau(25 \text{ km}) - 28 \text{ months}]^2}{[28 \text{ months}]^2} \quad (*)$$

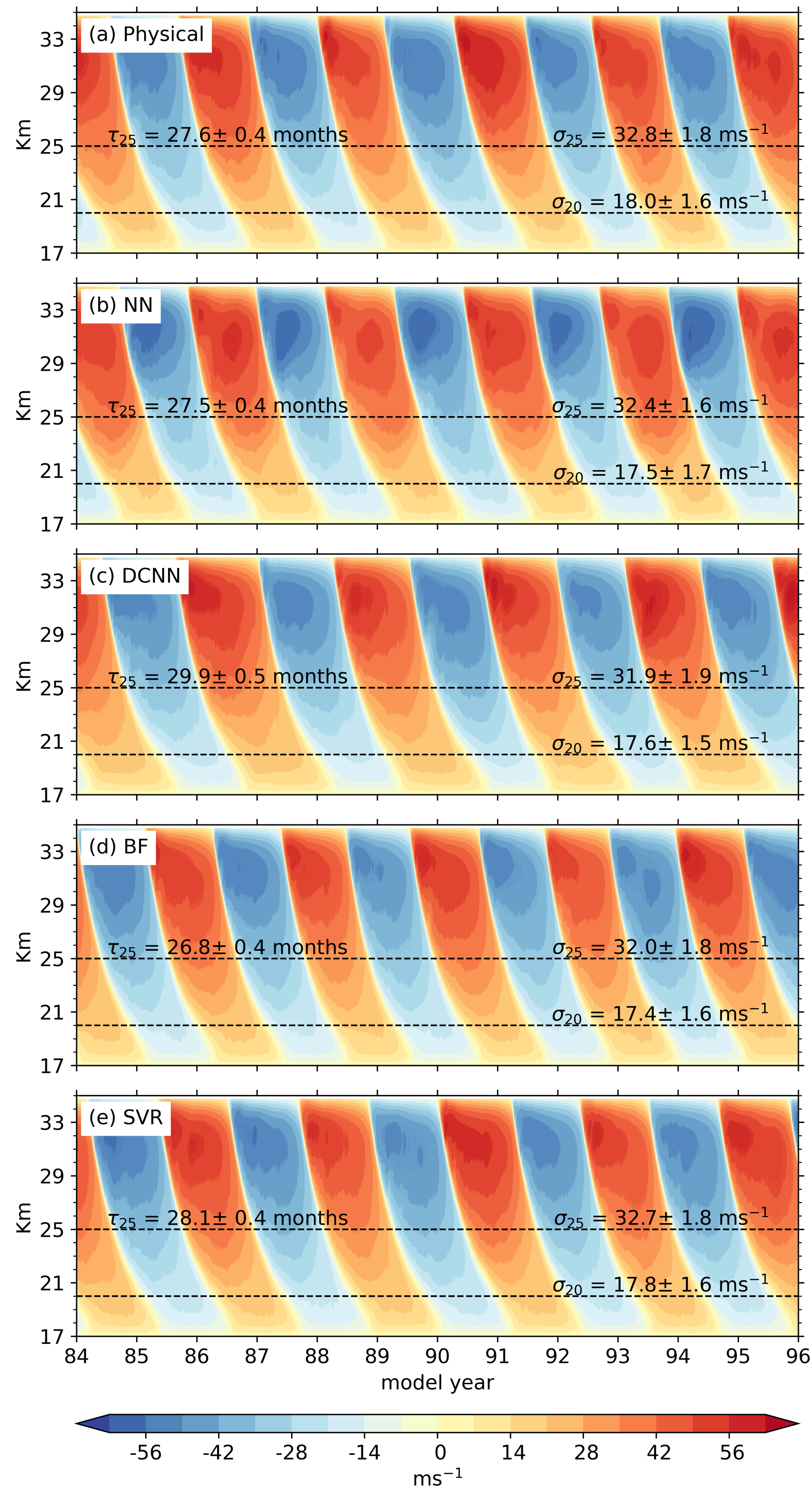


- The mean wave flux and spectral width required to capture the “observed” QBO in the 1D model are remarkably similar to those found in higher complexity models.

## (4) Emulation

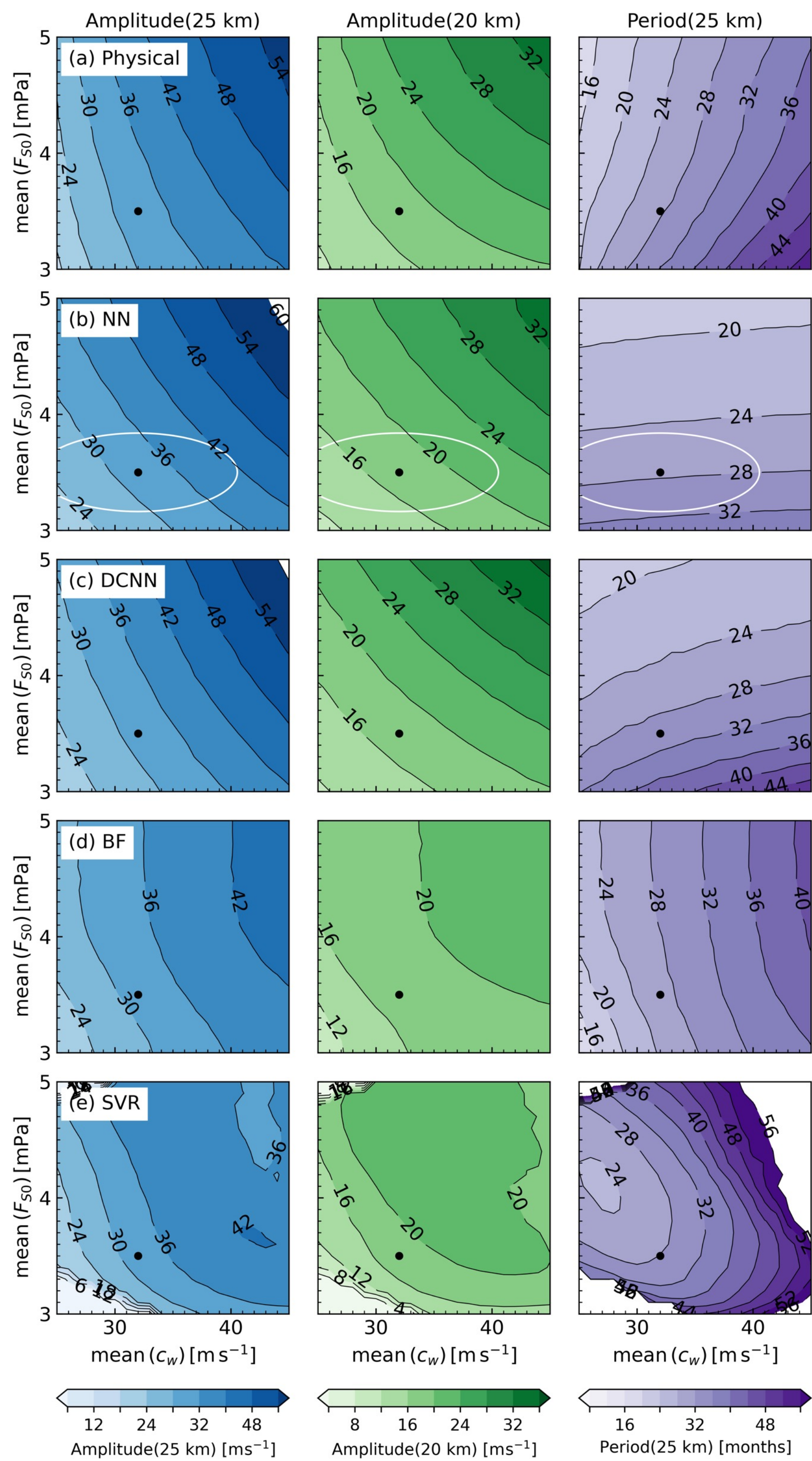
- We train different emulators using the “optimal” GW flux distribution for the training data.

$$-\frac{1}{\rho} \frac{\partial}{\partial z} \sum_i A_i \exp \left\{ -\int_{z=z_1}^z \frac{\alpha(z') N}{k_i (u - c_i)^2} dz' \right\} \rightarrow \text{Emulator}(u, F_{S0}, c_w)$$



## (5) Generalization

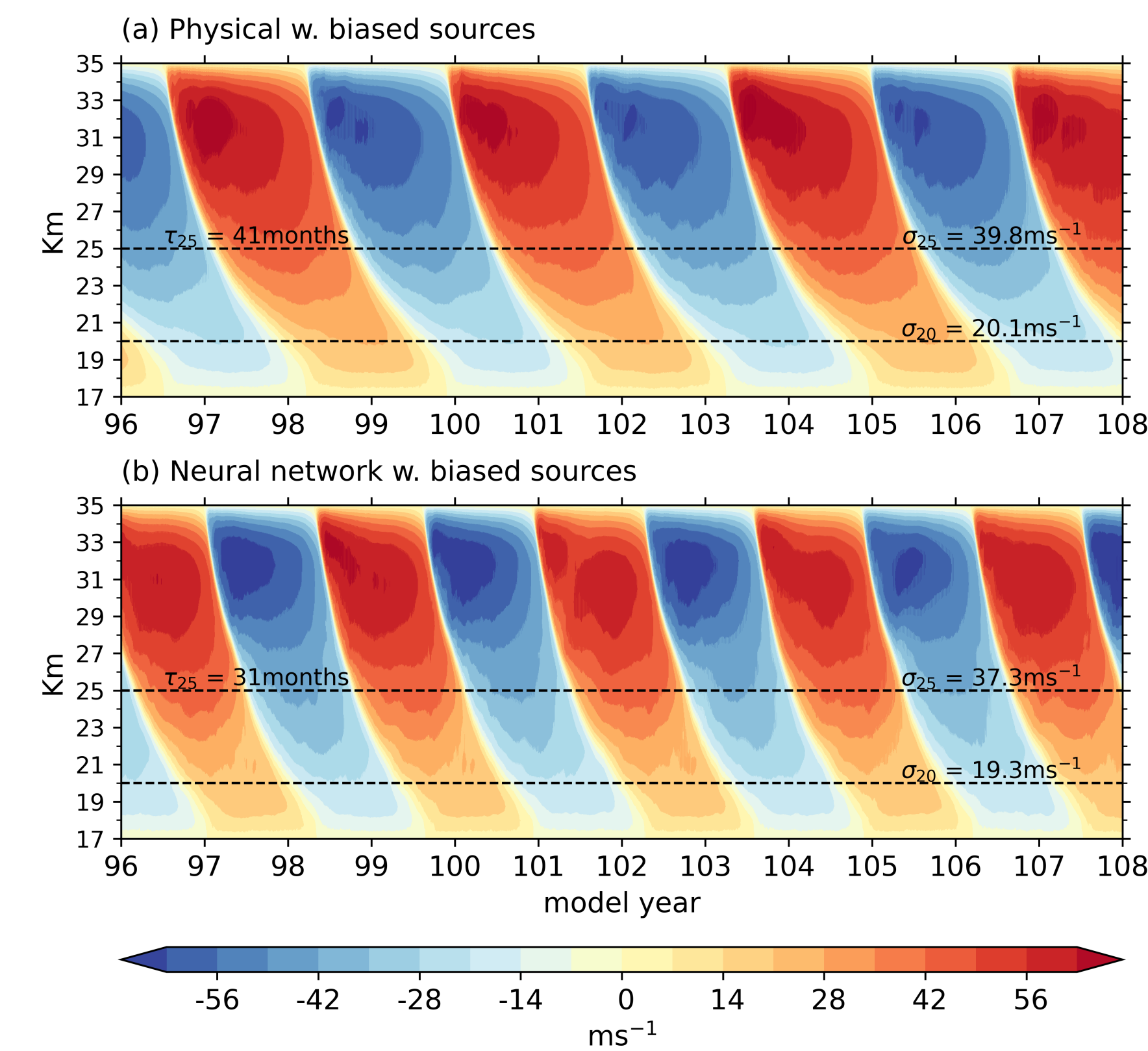
- How well do emulators trained on a single source distribution generalize to nearby source distributions?



- The emulators **capture** the qualitative sensitivity of the QBO's **amplitude** to changes in  $F_{S0}$  and  $c_w$ , but **struggle** to capture the qualitative sensitivity of the QBO's **period**.

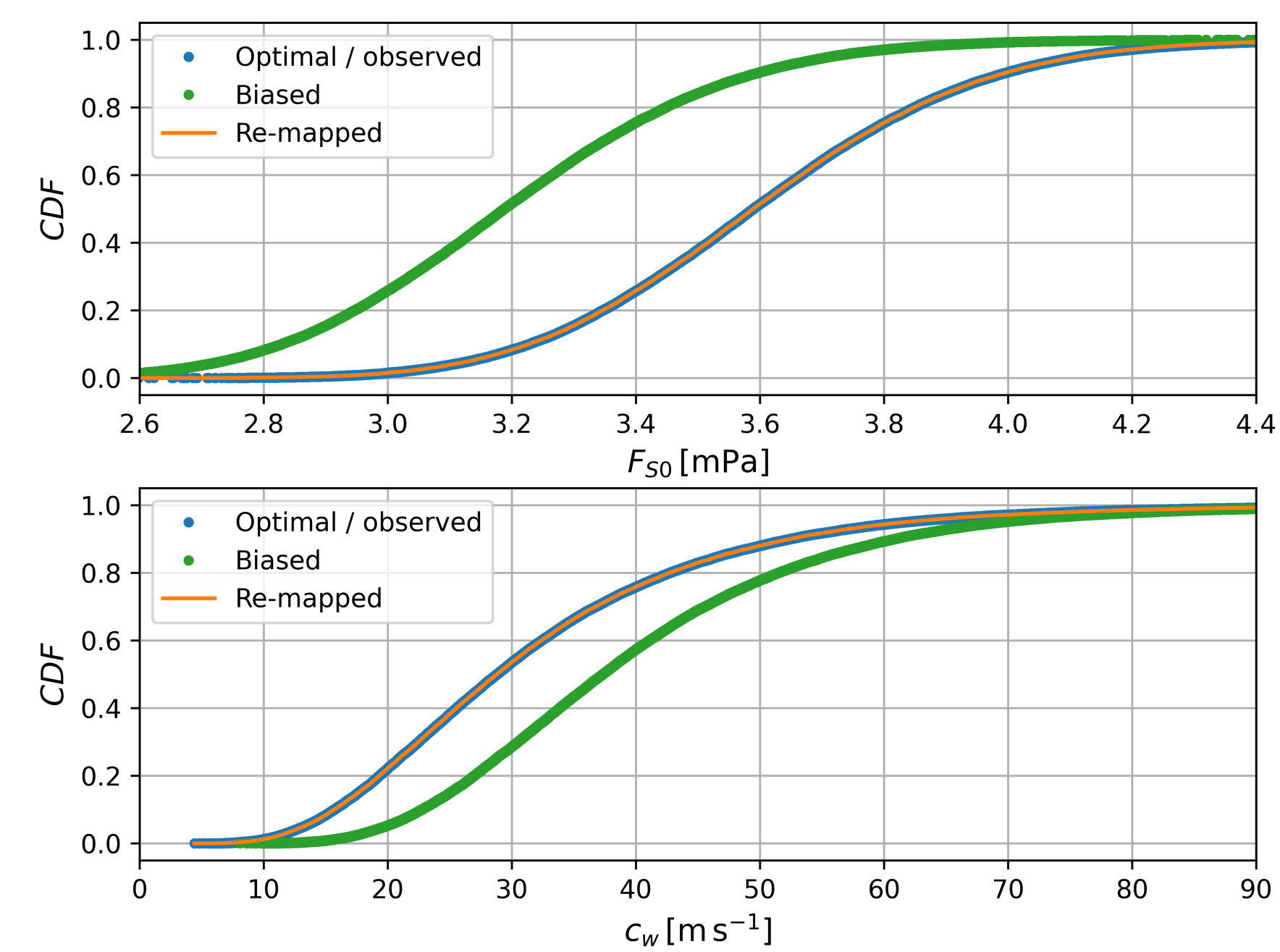
## (6) Calibration

- Our emulators were trained on the optimal/observed source distribution, but a model's source distribution can be biased.
- Consider the emulated solution forced by the biased source distribution indicated by black \* in box (3):

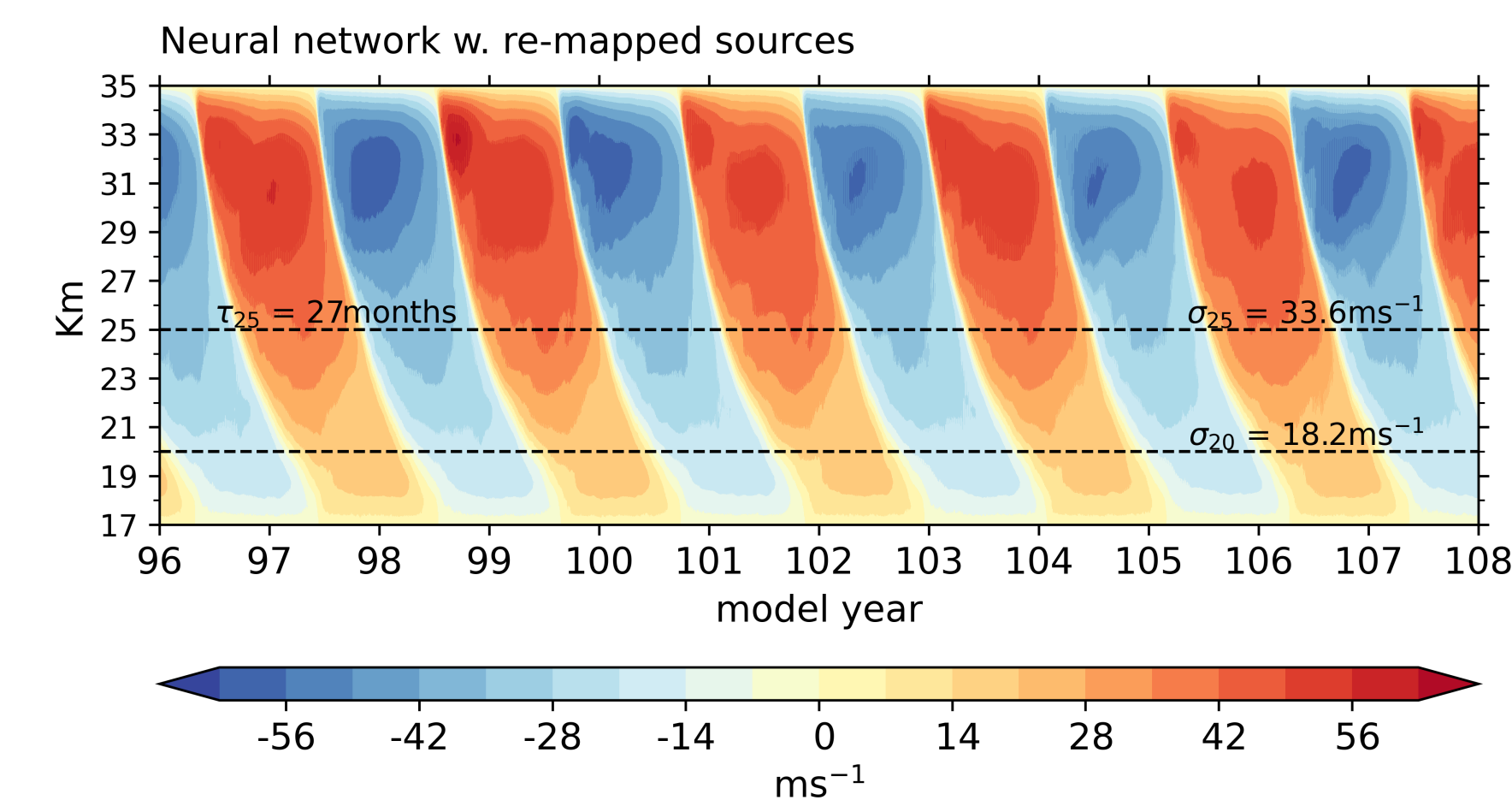


- How can we adjust the data-driven GW parameterization to yield the desired QBO statistics as in (\*)?
- One approach is to re-map the biased source distribution to the optimal source distribution:

$$\{F_{S0}, c_w\} \rightarrow CDF_{Optimal}^{-1}(CDF_{Biased}(\{F_{S0}, c_w\}))$$



- Yielding the desired QBO period and amplitudes:



## (7) Conclusions

- Machine learning methods show promise in replicating a physics based GWP, yielding stable, accurate simulations when coupled under the current climatological conditions.
- Our results demonstrate the challenge of generalizing to out of sample data, a major challenge of data-driven methods.
- Training on more complex data, having an annual cycle in  $w$ , does not improve our emulators' ability to generalize.
- If our emulators struggle to capture the sensitivity of the QBO's period to changes in the GW spectrum of a known physics-based parameterization, how well can we trust data-driven methods trained on observation to capture this sensitivity?

## References

<sup>1</sup>Alexander, M. J., & Dunkerton, T. J. (1999). A spectral parameterization of mean-flow forcing due to breaking gravity waves. *Journal of the Atmospheric Sciences*, 56(24), 4167-4182.

<sup>2</sup>Holton, J. R., & Lindzen, R. S. (1972). An updated theory for the quasi-biennial cycle of the tropical stratosphere. *Journal of Atmospheric Sciences*, 29(6), 1076-1080.

<sup>3</sup>Plumb, R. A. (1977). The interaction of two internal waves with the mean flow: Implications for the theory of the quasi-biennial oscillation. *Journal of Atmospheric Sciences*, 34(12), 1847-1858.

<sup>4</sup>Garfinkel, C. I., Gerber, E. P., Shamir, O., Rao, J., Jucker, M., White, I., & Paldor, N. (2022). A QBO Cookbook: Sensitivity of the Quasi-Biennial Oscillation to Resolution, Resolved Waves, and Parameterized Gravity Waves. *Journal of Advances in Modeling Earth Systems*, 14(3), e2021MS002568.