Supporting Information for “Machine Learning Gravity Wave Parameterization Generalizes to Capture the QBO and Response to Increased CO₂”
Zachary I. Espinosa¹, Aditi Sheshadri¹, Gerald R. Cain ², Edwin P. Gerber³,
Kevin J. DallaSanta⁴,⁵

¹Department of Earth System Science, Stanford University, Stanford, CA, USA
²Department of Computer Science, Stanford University, Stanford, CA, USA
³Courant Institute of Mathematical Sciences, New York University, New York, NY, USA
⁴NASA Goddard Institute for Space Studies, New York, NY, USA
⁵Universities Space Research Association, Columbia, MD, USA

Contents of this file
1. Table S1
2. Figures S1-S4

Additional Supporting Information (Files uploaded separately)
1. Captions for Movies S1-S4

Corresponding author: Zachary I. Espinosa, (zespinos@stanford.edu)
Introduction

The supporting information includes one table, four figures, and four movies. Table S1 shows a list of input variables accepted by the ANNs and output variables generated by the ANNs. Figure S1 provides a schematic and description of the ANN architectures. Figure S2 shows SHAP bar plots of the ten most important meridional and zonal wind features used by WaveNet to generate GWD at 10 hPa and 100 hPa. Figure S3 shows the difference in performance for WaveNet with 100K and 350K parameters for the first 30 simulation years in the same region as Figure 3. Figure S4 shows 15-day averaged zonal drag at 60N, corresponding to the winds associated with the polar vortex, for years 25-30 for AD99 and WaveNet in the control and 4xCO\textsubscript{2} simulations. Movies S1-S4 are time series animations of global zonal and meridional gravity wave drag generated by WaveNet, AD99, and their difference at 10 hPa and 100 hPa.

**Movie S1.** A latitude-longitude time series of zonal ANN predictions, AD99 truth, and their difference at model level 13 (10 hPa) for one year of test data.

**Movie S2.** A latitude-longitude time series of meridional ANN predictions, AD99 truth, and their difference at model level 13 (10 hPa) for one year of test data.

**Movie S3.** A latitude-longitude time series of zonal ANN predictions, AD99 truth, and their difference at model level 23 (100 hPa) for one year of test data.

**Movie S4.** A latitude-longitude time series of meridional ANN predictions, AD99 truth, and their difference at model level 23 (100 hPa) for one year of test data.

April 8, 2022, 5:28pm
Table S1. List of full resolved flow input variables accepted by the ANNs and output variables generated by the ANNs. The total input feature vector contains 203 elements, and the output is 33 GWD values. Two networks are trained, one for zonal drag and one for meridional drag.

<table>
<thead>
<tr>
<th>Input Variables</th>
<th>Vertical Levels</th>
<th>Output Variables</th>
<th>Vertical Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zonal Wind (m/s)</td>
<td>40</td>
<td>GW zonal drag (m/s)</td>
<td>33</td>
</tr>
<tr>
<td>Meridional Wind (m/s)</td>
<td>40</td>
<td>GW meridional drag (m/s)</td>
<td>33</td>
</tr>
<tr>
<td>Vertical Wind (m/s)</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature (K)</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height (m)</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latitude (λ)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitude (ϕ)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface Pressure (hPa)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of Stacked Array</td>
<td>203</td>
<td></td>
<td>33</td>
</tr>
</tbody>
</table>
WaveNet contains 4 shared hidden layers, each with 256 neurons. WaveNet then splits into 33 branches (one branch per nontrivial vertical layer) each containing 4 pressure level specific layers with 256, 128, 64, and 32 neurons, respectively. Each branch then outputs a single value corresponding to the gravity wave drag at that vertical layer. While the architecture remains constant, the exact number of shared hidden layers, pressure level layers, and neurons varies with the size and input of WaveNet. The numbers provided are for the largest ANN which has 3,848,481 parameters and uses the total input feature vector.
Figure S2. Panels a) through d) are SHAP bar plots of the ten most important meridional (a,b) and zonal (c,d) wind features used by WaveNet to generate meridional or zonal GWD at 10 hPa and 100 hPa when trained on the full feature set. The vertical axes’ values indicate displacements in vertical pressure levels, with positive and negative values being above and below the level of prediction, respectively (e.g., u -1 indicates zonal wind at the vertical level directly below the level of prediction). The results suggest that vertically local wind fields are the dominant features used by WaveNet to generate GWD.
Figure S3. Pressure-Time profiles of the zonal mean zonal wind, averaged from between 5 S and 5 N and smoothed with a 15-day low pass filter show the WaveNet 350k and WaveNet 100k parameter generated QBO for the first 30 years of integration of (a,b) the control version of MiMA and (c,d) 4xCO₂; the 4xCO₂ integrations include the initial period where the surface temperature adjusts to the new radiative forcing. Vertical dashed lines separate five year segments. The dashed-horizontal line in each panel delineates the model level (≈10 hPa) where the TT method is used. Unlike for the 350k version, the period of the QBO generated by WaveNet 100k in the control experiment is too long compared to MiMA with AD99.
Figure S4. Pressure-Time profiles of 15-day averaged zonal winds at 60N latitude for years 25-30 for AD99 (a,c) and WaveNet (b,d) in the control version of MiMA (a,c) and 4xCO$_2$ (b,d) 30-year simulations. Vertical dashed lines separate one year segments. The westerlies (red) and easterlies (blue) bands correspond to winds associated with the seasonal cycle of the polar vortex.