Atmospheric dynamics assignments

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1 Radiative Equilibrium of a Gray Atmosphere

Compute a "gray atmosphere" radiative equilibrium solution for an Earth-like planet with a 1000 hPa atmosphere, emission temperature of $T_e = 255$ K, and surface temperature of $T_s = 288$ K. We'll assume a 1-D profile that represents the whole planet. To do this, you'll need find the correct τ_∞ to get the right greenhouse effect.

To plot this in pressure or height, you will also need to find the right κ to map this to an atmosphere with a total pressure of 1000 hPa. We will assume a uniformly mixed greenhouse gas without any pressure broadening. This step is not explicitly needed for the radiative equilibrium solution, but will be necessary when you consider radiative convective equilibrium. It makes it easier to interpret if you can plot $T(z)$.

What would a solution look like? Recall that radiative transfer through your atmosphere satisfies these equations:

$$\begin{aligned} \frac{dI_+}{d\tau} &= -I_+(\tau) + \sigma T(\tau)^4 \\ \frac{dI_-}{d\tau} &= I_-(\tau) - \sigma T(\tau)^4 \end{aligned}$$

with the boundary conditions $I_+(0) = \sigma T_s^4$ and $I_-(\tau_\infty) = 0$, that is, the surface emits as a blackbody with temperature T_s and there is no downwelling terrestrial radiation at the top of the atmosphere. These equations assume you have a temperature profile $T(\tau)$. It is your goal to find the equilibrium profile $T(\tau)$ so that radiative transfer is divergence free (no heating or cooling) *and* the surface is equilibrium at the same time, satisfying

$$\frac{S_0(1-\alpha)}{4} + I_-(0) = \sigma T_s^4. \tag{1}$$

where $S_0(1-\alpha)/4 = T_e^4 = 239$ W/m^2.

One approach is to assume a value of τ_∞ (say, $\tau_\infty = 1$) and compute the equilibrium surface temperature T_s associated with it. Then you need to increase/decrease τ_∞ to get $T_s = 288$ K.

Another approach (thanks Gabriel!), would be to assume a value of T_s and τ_∞, and then check if your surface energy balance is satisfied. If you start with our target $T_s = 288$, then you know the RHS of (1). If the LHS is greater, than τ_∞ is too large, and you need to reduce it. If the LHS is smaller, τ_∞ needs to be increased.

1.1 An iterative approach

One approach is to set up an iterative model, where you assume some initial profile of temperature, and then step forward in time until it converges. At the surface,

$$\frac{S_0(1-\alpha)}{4} + I_-(0) = \sigma T_s^4 + c_s \frac{dT_s}{dt}$$

For the atmosphere, the radiative heating is given by the divergence of the radiative fluxes:

$$h = -\frac{d}{d\tau}(I_+ - I_)$$

Assuming some heat capacity for the atmosphere, say c_τ, where the τ subscript emphasizes this is in optical thickness space, your atmosphere evolves as:

$$\frac{d}{dt}T(\tau) = \frac{h}{c_\tau}.$$

The devil will be in the details, getting values of c_s and c_τ so that you can stably intergrate the model, but also don't have to wear out your CPU by converging very slowly.

Gabriel's approach avoided having to time step the surface energy balance. Rather, you just allow T_s to set your lower boundary condition, interate just your atmosphere to equilibrium, and check if equation (1) is satisfied.