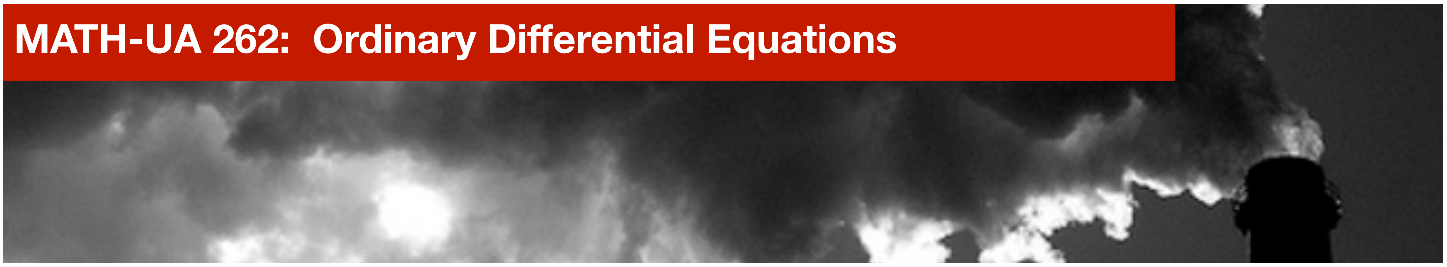
MATH-UA 262: Ordinary Differential Equations



Lectures: Monday and Wednesday, 9:30-10:45 am (WWH 512)

Edwin Gerber (epg2@nyu.edu)
Virtual Office Hours: Monday 1:30-2:30 pm, Wednesday 11am-12 noon
Zoom links are provided on our Bright Space page.

Recitation: Friday, 9:30-10:45 am (194 Mercer, Room 307)

Michael Sheng (ts4196@nyu.edu)
Virtual Office Hours: Thursday 1-2pm and Friday noon-1pm (links on Bright Space)

Textbook

Braun, Martin, *Differential Equations and their Applications, 4th edition*, Springer, 1993. [**free!** https://link.springer.com/book/10.1007%2F978-1-4612-4360-1]

Course Description

Ordinary Differential Equations, ODE for short, was probably my least favorite class in mathematics when I was an undergraduate. It may have something to do with the fact that the class met at 8 am, or that we used a book written by the professor, but at the time I felt that there were so few equations we could solve (at least out of the universe of equations one could pose), and these solutions often depended on a few tricks, rather than something fundamental. Ugh, I really missed the point!

ODE is where you start to see the power of calculus, *the ability to make predictions*. Yes, with calculus you can predict the future! Well, given the physical laws of the universe (or the markets, or your favorite system), *differential equations* allow us to forecast how it will evolve. They are indispensable in my field of research, where we work with ODE's complicated sibling (*partial differential equations*, which involve multiple dimensions) to predict the response of the climate systems to external forcing, i.e., our greenhouse gas emissions. They are invaluable across the sciences.

ODE also allows us to ask fundamental questions about equations. Do solutions exist (i.e., will this universe end)? Are solutions unique, or are parallel universes possible next to each other?!? In this, it offers a glimpse at the field of analysis.

The (more modest) goals of this course are to cover these topics:

- methods for solving the (alas) few types of linear first and second order equations that can be solved exactly
- methods for proving existence and uniqueness of solutions
- series solutions for equations with singular points
- systems of linear equations
- nonlinear dynamical systems and phase plane analysis
- boundary value problems
- Green's functions and Fourier series. (If time allows.)

Steven Strogatz has eloquently argued that the point of calculus is to make hard problems easier. This is admittedly not how most students feel about the subject. Part of the problem is that many of the introductory courses are about learning the "grammar" of the subject. You can't write a novel until you've mastered the basics. The other problem, however, is that you're coming at it from a class. I (the instructor) assign you a seemingly arbitrary problems that you're supposed to solve for a grade. *But calculus is actually there to help us solve real world problems.*

Consider weather prediction. Just a few decades ago, weather forecasting was impossible. The width of the brown stripe on fuzzy caterpillars, or whether a marmot saw his shadow, was as good a prediction about the future as from a most learned scientist. Unless a ship happened to weather an approaching hurricane, storms simply slammed in unannounced. Similarly, there was little hope at predicting bomb cyclones, blizzards, or severe thunderstorms. It is calculus that made it possible to accurately forecast severe storms days in advance. Well, it was calculus and the ability to represent it on computers (which had to be invented, too!), and the dramatic increase in our ability to observe the atmosphere that really made it possible. We will touch on some of these topics in the class; I'll do my best to introduce examples from the real world in this course, but please bear with me. You are unfortunately still learning the grammar, and it my hope that in you will be able to use these skills to do great things in the future!

Resilience and Safety

Unfortunately this term is unlikely to be ordinary, despite the title of the course. In keeping with University policy, I will not require attendance — **please stay home if you are feeling ill or otherwise believe you may pose any risk to others**. When in doubt, please play it safe. ODE can wait!

To help students who are quarantining or otherwise unable to come in-person, I will will share my lecture notes and offer virtual office hours. Please contact me on an individual basis if you expect to miss more than a couple classes due to illness.

And remember, always wear a mask in class — I'll show you equations that demonstrate why it matters!

Homework and Exams

Homework will be due each Friday. Last semester I found that submitting it online was the preferred option, but if you'd like to submit physical paper, let me know and we can make arrangements. In fairness to the grader (and other students), **late homework will not be accepted.** Your worst two scores (or missed assignments) will automatically be dropped to account for weeks when you were under the weather or had to focus on other courses, etc.. Note that the homework is worth as much as the final, and that the final is only 30% of the grade. The points for homework is partly to let you bank points before the exams, but mainly to incentivize the homework: you will learn ODE by solving ODEs, more than listening to my lectures. The lectures and text are there to give you the tools and strategies, but it's putting pencil to paper (or stylus to tablet) that you'll really master the topic.

Did I mention that late homework won't be accepted? I don't mean to be a jerk about this, but experience has shown that a strict deadline is the only fair way to proceed. And once you are out in the real world and the president needs the answer by 5 pm, you will want to give it to her at 5 pm, not 5:30, or the next day, even if your best friend was visiting, etc. And the good news is that you get to drop two homework. And beyond that, this is just undergraduate ODE. No one will lose their job or die if you make a mistake here. College is a much better place to mistakes than the real world!

The remainder of your grade will be determined by two midterm exams (2.16 and 4.13) and a cumulative final exam. The early first midterm is to give you a chance to see where you are in the course sooner rather than later.

And finally, a word on academic integrity. D**o not cheat.** I will require an academic pledge on all exams. If you work with others on homework (which I encourage), please just note your partners. **If you are caught cheating, the penalty will be a zero for the assignment/exam, failing the course, and/or reporting you to the Office of the Dean, depending on the severity of the infraction.**

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. The Moses Center website is www.nyu.edu/csd. Please contact the Moses Center for Students with Disabilities (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.

Grade Policy

Weekly Homework	30%
Midterm 1	20%
Midterm 2	20%
Final Exam	30%

Tentative Course Schedule (this is admittedly ambitious!)

Week	Topics	Readings
1.24, 1.26	Introduction: R_0 and herd immunity; First order linear equations	1.1 (and 4.12), 1.2
1.31, 2.2	separation of variables, population models	1.4, 1.5
2.7, 2.9	existence and uniqueness	1.9, 1.10
2.14, 2.16	Numerical Solutions, **Midterm I**	1.13, 1.16
2.22	Online lecture: Second order linear equations	2.1
2.28, 3.2	Second order linear eqns with (non)constant coefficients, (in)homogenous, variation of parameters.	2.2-5
3.7, 3.9	Oscillator problems, series solutions	2.6,8
3.21, 3.23	Singular points, method of Froebenius, special functions	2.8
3.28, 3.30	Systems of linear equations, eigenvector methods	3.1, 3.8
4.4, 4.6	Systems, continued; complex roots, equal roots, fundamental solution, metric exponentiation	3.9-12
4.11, 4.13	Nonlinear autonomous equations, fixed points and linearization, stability. **Midterm II**	4.1-2
4.18, 4.20	Stability, Phase plan analysis, Predator-prey, the SIR model, revisited	4.3-7 , 4.10, 4.12
4.25, 4.27	Boundary value problems, Sturm-Louiville	5.1, 6.3-4
5.2, 5.4	Heat equation, Fourier series	5.2-6
5.9	Dirac delta-functions and Green's Functions	2.12-13