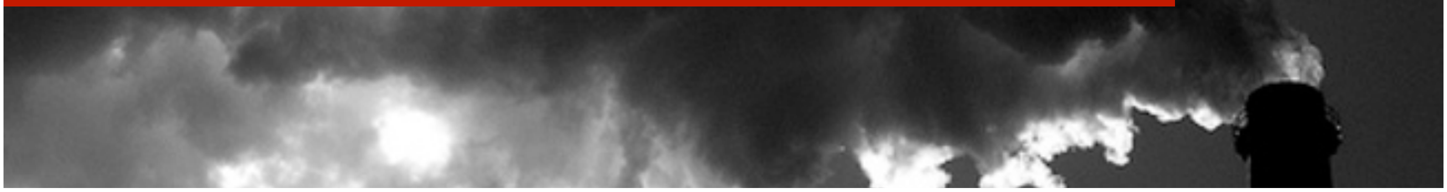
MATH-UA 262: Ordinary Differential Equations



Lectures: Monday and Wednesday, 2:00 - 3:15 pm (Kimmel 914)

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Friday 3:30 - 4:45 pm (Silver 405)

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Office Hours: Tuesday and Thursday 11 am - 12 noon (Warren Weaver 609)

Textbook

Braun, Martin, *Differential Equations and their Applications, 4th edition*, Springer, 1993. [**free!** https://link.springer.com/book/10.1007%2F978-1-4612-4360-1]

Course Description

Ordinary Differential Equations, ODE for short, was probably my least favorite class in mathematics when I was an undergraduate. It may have something to do with the fact that the class met at 8 am, and that we used a book written by the professor, but at the time I felt that it seemed there were so few equations we could solve (at least out of the universe of equations one could pose), and these solutions often depended on a few tricks, rather than something fundamental. Ugh, I really missed the point!

ODE is where you start to see the power of calculus, *the ability to make predictions*. Yes, with calculus you can predict the future! Well, more precisely, given the physical laws of the universe (or the markets, or your favorite system), *differential equations* allow us to forecast how it will evolve into the future. They are indispensable in my field of research, where we work with ODE's complicated sibling (*partial differential equations*, which involve multiple dimensions) to predict the response of the weather and climate systems to external forcing, i.e., our greenhouse gas emissions. They are invaluable across the physical sciences.

ODE also allows us to ask fundamental questions about equations. Do solutions exist (i.e., will this universe end suddenly?) Are solutions unique, or are parallel universes possible next to each other?!? In this, it offers a glimpse at the field of analysis.

The (more modest) goals of this course are to cover these topics:

- methods for solving the (alas) few types of linear first and second order equations that can be solved exactly
- methods for proving existence and uniqueness of solutions
- series solutions for equations with singular points
- systems of linear equations
- nonlinear dynamical systems and phase plane analysis

- boundary value problems
- Green's functions and Fourier series.

Steven Strogatz has eloquently argued that the point of calculus is to make hard problems easier. This is admittedly not how most students feel about the subject. Part of the problem is that many of the introductory courses are about learning the "grammar" of the subject. You can't write a novel until you've mastered the basics. The other problem, however, is that you're coming at it from a class. I (the instructor) assign you a seemingly arbitrary problems that you're supposed to solve for a grade. *But calculus is actually there to help us solve real world problems.*

Consider weather prediction. Just a few decades ago, weather forecasting was impossible. The width of the brown stripe on fuzzy caterpillars, or whether a marmot saw his shadow, was as good a prediction about the future as from a most learned scientist. Unless a ship happened to weather an approaching hurricane, storms simply slammed in unannounced. Similarly, there was little hope at predicting bomb cyclones, blizzards, or severe thunderstorms. It is calculus that made it possible to accurately forecast severe storms days in advance. Well, it was calculus and the ability to represent it on computers (which had to be invented, too!), and the dramatic increase in our ability to observe the atmosphere that really made it possible. We will touch on some of these topics in the class; I'll do my best to introduce examples from the real world in this course, but please bear with me. You are unfortunately still learning the grammar, and it my hope that in you will be able to use these skills to do great things in the future!

Class Expectations and Safety

Unfortunately this term is unlikely to be ordinary, despite the title of the course. The plan is to return to in-person instruction, but with vaccinations and masks to protect those who cannot be vaccinated (including children and those with medical conditions). We will need to be ready for things change rather abruptly. In keeping with University policy, I will not require attendance — **please do stay home if you are feeling ill or otherwise believe you may pose any risk to others**. Between the text book, notes that I will share with you, office hours offered by Luke and myself, and your fellow classmates, you can surely get caught up when you are in the clear. When in doubt, please play it safe. ODE can wait!

Homework and Exams

Homework will be due each Thursday. It must be in my mailbox in Warren Weaver by first thing Friday morning. In fairness to the graders (and other students), **late homework will not be accepted.** Your worst two scores (or missed assignments) will automatically be dropped to account for weeks when you were under the weather or had to focus on other courses, etc.. Note that the homework is worth as much as the final, and that the final is only 30% of the grade. The points for homework is partly to let you bank points before the exams, but mainly to incentivize the homework: you will learn ODE by solving ODEs, much more than listening to my lectures. The lectures and text are there to give you the tools and strategies, but it's putting pencil to paper that you'll really master the topic.

Did I mention that late homework won't be accepted? I don't mean to be a jerk about this, but experience has shown that a strict deadline is the only fair way to proceed. And once you are out in the real world and the president needs the answer by 5 pm, you will want to give it to her at 5 pm, not 5:30, or the next day, even if your best friend was visiting, etc. And the good news is that you get to drop two homework. And beyond that, this is just undergraduate ODE. No one will die if you make a mistake here. College is a much better place to mistakes than the real world!

The remainder of your grade will be determined by two midterm exams (tentatively set to 10/6 and 11/17) and a cumulative final exam. The somewhat early midterm is to give you a chance to see where you are in the course sooner rather than later.

And finally, a word on academic integrity. **Please do not cheat.** I will require an academic pledge on all exams. If you work with others on homework (which I encourage), please just note your partners. **If you are caught cheating, the penalty will be a zero for the assignment/exam, failing the course, and/or reporting you to the Office of the Dean, depending on the severity of the infraction.**

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. The Moses Center website is www.nyu.edu/csd. Please contact the Moses Center for Students with Disabilities (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.

Grade Policy

Weekly Homework	30%
Midterm 1	20%
Midterm 2	20%
Final Exam	30%

Tentative Course Schedule (this is admittedly ambitious!)

Week	Topics	Readings
Sept 8	Introduction: R_0 and herd immunity	1.1 (and 4.12)
Sept 13,15	First order linear equations, separation of variables	1.2,1.4
Sept 20,22	population models, existence	1.5, 1.9
Sept 27,29	Uniqueness, Numerical Solutions	1.10, 1.13, 1.16
Oct 4,6	Second order linear equations. **Midterm I**	2.1
Oct **12**,13	Second order linear eqns with constant coefficients, homogenous and inhomogeneous, damped oscillator and resonance. (Tuesday 10/12 is a Monday at NYU!)	2.2-3, 2.5-6
Oct 18,20	Second order linear equations with non-constant coefficients, variation of parameters, series solutions	2.4-5
Oct 25,27	Singular points, method of Froebenius, special functions	2.8
Nov 1,3	Systems of linear equations, eigenvector methods	3.1, 3.8
Nov 8,10	Systems, continued; complex roots, equal roots, fundamental solution, metric exponentiation	3.9-12
Nov 15,17	Nonlinear autonomous equations, fixed points and linearization, stability. **Midterm II**	4.1-2
Nov 22,24	Stability, Phase plan analysis, Predator-prey, the SIR model, revisited	4.3-7 , 4.10, 4.12
Nov 29, Dec 2	Boundary value problems, Sturm-Louiville	5.1, 6.3-4
Dec 6, 8	Heat equation, Fourier series	5.2-6
Dec 13	Dirac delta-functions and Green's Functions	2.12-13