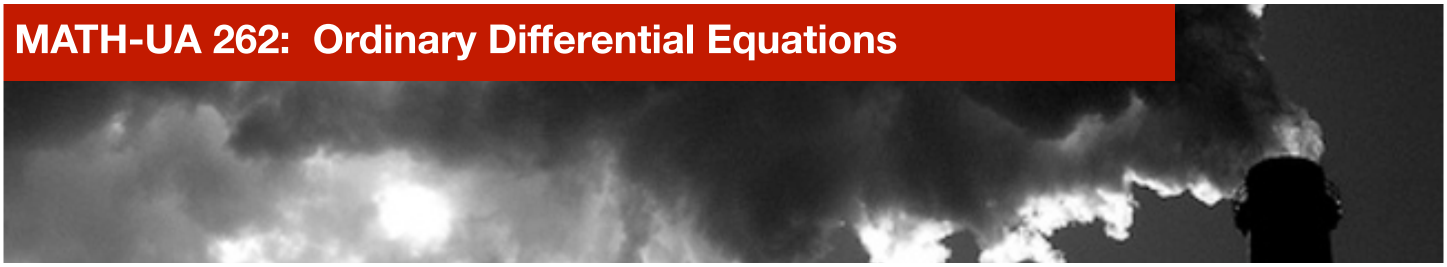
MATH-UA 262: Ordinary Differential Equations



Lectures: Monday and Wednesday, 2:00 - 3:15 pm (remote)

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Recitation: Friday 11 am - 12:15 pm (Silver 405 and remote)

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Textbook

Braun, Martin, *Differential Equations and their Applications, 4th edition*, Springer, 1993. [free! https://link.springer.com/book/10.1007%2F978-1-4612-4360-1]

Course Description

Ordinary Differential Equations, ODE for short, was probably my least favorite class in mathematics when I was an undergraduate. It may have something to do with the fact that the class met at 8 am, and that we used a book written by the professor, but at the time I felt that it seemed there were so few equations we could solve (at least out of the universe of equations one could pose), and these solutions often depended on a few tricks, rather than something fundamental. Ugh, I really missed the point!

ODE is where you start to see the power of calculus, *the ability to make predictions*. Yes, with calculus you can predict the future! Well, more precisely, given the physical laws of the universe (or the markets, or your favorite system), *differential equations* allow us to forecast how it will evolve into the future. They are indispensable in my field of research, where we work with ODE's complicated sibling (*partial differential equations*, which involve multiple dimensions) to predict the response of the weather and climate systems to external forcing, i.e., our greenhouse gas emissions. They are invaluable across the physical sciences.

ODE also allows us to ask fundamental questions about equations. Do solutions exist (i.e., will this universe end suddenly?) Are solutions unique, or are parallel universes possible next to each other?!? In this, it offers a glimpse at the field of analysis.

The (more modest) goals of this course are to cover these topics:

- methods for solving the (alas) few types of linear first and second order equations that can be solved exactly
- methods for proving existence and uniqueness of solutions
- series solutions for equations with singular points
- systems of linear equations
- nonlinear dynamical systems and phase plane analysis
- boundary value problems
- Green's functions and Fourier series.

Steven Strogatz has eloquently argued that the point of calculus is to make hard problems easier. This is admittedly not how most students feel about the subject. Part of the problem is that many of the introductory courses are about learning the "grammar" of the subject. You can't write a novel until you've mastered the basics. The other problem, however, is that you're coming at it from a class. I (the instructor) assign you a seemingly arbitrary problems that you're supposed to solve for a grade. *But calculus is actually there to help us solve real world problems.*

Consider weather prediction. Just a few decades ago, weather forecasting was impossible. The width of the brown stripe on fuzzy caterpillars, or whether a marmot saw his shadow, was as good a prediction about the future as from a most learned scientist. Unless a ship happened to weather an approaching hurricane, storms simply slammed in unannounced. Similarly, there was little hope at predicting bomb cyclones, blizzards, or severe thunderstorms. It is calculus that made it possible to accurately forecast severe storms days in advance. Well, it was calculus and the ability to represent it on computers (which had to be invented, too!), and the dramatic increase in our ability to observe the atmosphere that really made it possible. We will touch on some of these topics in the class; I'll do my best to introduce examples from the real world in this course, but please bear with me. You are unfortunately still learning the grammar, and it my hope that in you will be able to use these skills to do great things in the future!

Class Expectations

Unfortunately this term will be anything but ordinary. We are meeting remotely, physically separated from each other and subject to the vagaries of the internet. Moreover we are spread out in various time zones across the world, making it all but impossible for all of us to meet at the same time. Given these impediments, I'll employ a so-called "flipped classroom" format, where lectures will be pre-recorded and the class time will be used to go over the concepts and discuss problems. Class time meetings will also be recorded for those unable to attend.

To try our best to overcome our physical separation, we'll be using a buffet of technology: NYU Classes (for organization and communication), Zoom (lectures and office hours), CampusWire (communication), and Microsoft OneNote (for notes, as the name might imply). Our Classes site will be the central depot, look for details there.

I appreciate that the physical separation also increases the temptation to cheat on exams and other assignments. **Please do not cheat.** I will require an academic pledge on all exams. If you work with others on homework (which I encourage), please just note your partners. **If you are caught cheating, the penalty will be a zero for the assignment/exam, failing the course, and/or reporting you to the Office of the Dean, depending on the severity of the infraction.**

Here's a sketch of the work flow in our course.

Before class: Before each meeting, please read the required sections of the text book and view the formal lecture (via our NYU Classes page). The notes will be shared as well (Microsoft OneNote). Small quizzes and/or lead off questions will periodically help highlight important concepts from the reading and lecture. I encourage you to raise questions on Campuswire before the class meetings, particularly if you will be unable to attend due to time zone conflicts.

During class time: we'll be using Zoom and Campuswire to collaborate on problems. Breakout sessions will also allow us to work in small groups. I encourage you to set up your own meetings with classmates outside of class time to work and learn together, especially if the time shift prevents you from participating during regular class time

After class: Homework will be due each Wednesday by 23:59 New York time. In fairness to the graders (and other students), late homework will not be accepted. Your worst score (or a missed assignment) will automatically be dropped. Note that the homework and quizzes are worth as much as the final, and that the final is only 30% of the grade.

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. The Moses Center website is www.nyu.edu/csd. Please contact the Moses Center for Students with Disabilities (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.

Grade Policy

Weekly Homework and Quizzes	30%
Midterm 1	20%
Midterm 2	20%
Final Exam	30%

Tentative Course Schedule

Week	Topics	Readings
Sept 2	Introduction: R_0 and herd immunity	1.1 (and 4.12)
Sept 9	Classification of ODEs, First order linear equations	1.2
Sept 14,16	Separation of variables, population models	1.4-5
Sept 21,23	Existence, Uniqueness	1.9-10
Sept 28,30	Numerical Solutions (Euler and Runge-Kutta), Second order linear equations	1.13, 1.16, 2.1
Oct 5,7	Second order linear eqns with constant coefficients, homogenous and inhomogeneous, damped oscillator and resonance.	2.2-3, 2.5-6
Oct 12,14	Second order linear equations with non-constant coefficients, variation of parameters, series solutions	2.4-5
Oct 19,21	Singular points, method of Froebenius, special functions	2.8
Oct 26,28	Systems of linear equations, eigenvector methods	3.1, 3.8
Nov 2,4	Systems, continued; complex roots, equal roots, fundamental solution, metric exponentiation	3.9-12
Nov 9,11	Nonlinear autonomous equations, fixed points and linearization, stability.	4.1-2
Nov 16,18	Stability, Phase plan analysis, Predator-prey, the SIR model, revisited	4.3-7 , 4.10, 4.12
Nov 23,25	Boundary value problems, Sturm-Louiville	5.1, 6.3-4
Nov 30, Dec 2	Heat equation, Fourier series	5.2-6
Dec 7, 9	Dirac delta-functions and Green's Functions	2.12-13